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THESIS

OPTIMIZING EMPLOYMENT OF SEARCH PLATFORMS TO COUNTER SELF-PROPELLED SEMI-SUBMERSIBLES

by

Daniel M. Pfeiff

June 2009

Thesis Advisor:	Gerald G. Brown
Second Reader:	Jeffrey E. Kline

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13. ABSTRACT (maximum 200 words)

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OPTIMIZING EMPLOYMENT OF SEARCH PLATFORMS TO COUNTER SELF-PROPELLED SEMI-SUBMERSIBLES

Daniel M. Pfeiff
Commander, United States Navy
B.S., University of Kansas, 1990

Submitted in partial fulfillment of the
requirements for the degree of

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June 2009

Author: Daniel M. Pfeiff

Approved by: Distinguished Professor Gerald G. Brown
Thesis Advisor

Captain Jeffrey E. Kline, USN (RET)
Second Reader

Robert F. Dell
Chairman, Department of Operations Research

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ABSTRACT

Self-Propelled Semi-Submersibles now transport an estimated 75% of cocaine originating from Colombia and headed for the United States. There are several types of search platforms (i.e., units to detect, classify, and interdict) being employed by the Joint Interagency Task Force South to combat the semi-submersibles. We use a defender-attacker optimization model to maximize the defender's probability of successful detection and classification of the semi-submersible through the advantageous disposition of these search platforms against an intelligent attacker operating the semi-submersible. We assume the attacker has imperfect knowledge of defender platform disposition but is aware that there are defenders that must be avoided. Given this assumption, the solution to the defender-attacker model is a mixed (i.e., probabilistic) strategy for the defender and a least-risk path for the attacker. We demonstrate our defender-attacker model with both an Eastern Pacific and a Caribbean scenario using five representative search platform types whose detection and classification performance vary by platform, and by geography. In each of these cases, we find that our model prescribes a face-valid defensive plan; defenders take advantage of geography by positioning at chokepoints in constrained waterways, and they provide coverage near attacker origins and destinations in the less geographically-constrained scenarios.

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EXECUTIVE SUMMARY

The Self Propelled Semi-Submersible (SPSS) is not a true submarine, but a surface vessel with very low freeboard. Maritime drug traffickers in the Caribbean and Eastern Pacific have historically used a combination of commercial fishing vessels and smaller, harder-to-detect “go fast” boats. The SPSS combines the low profile of a “go-fast” boat with the long range and high capacity of a fishing boat. Given these advantages the SPSS has become an increasingly popular choice for smugglers trafficking drugs from Columbia to the Pacific Coast of Mexico. SPSSs now account for up to 75% of the estimated 600 tons of cocaine originating from Colombia annually. It is not lost on us that an SPSS can also carry passengers, or even a weapon of mass destruction.

There are several types of search platforms (i.e., units to detect, classify, and interdict SPSSs) being employed by the Joint Interagency Task Force (JIATF) South to combat the SPSS. These include surface ships and aircraft from several partner nations and U.S. agencies. We also consider employing non-traditional stationary acoustic sensors. Submarines are worth considering given the SPSSs relatively loud acoustic signature. Though a variety of platforms that can be used against the SPSS, there are generally only a few available at any given time.

Because searching for a single SPSS can tie up many search assets, current SPSS interdiction relies heavily on actionable prior intelligence. In the absence of prior intelligence operational commanders rely on information such as cones of courses and speed versus time with consideration for background effects such as weather to help determine the most likely routes for the SPSS. There is no computer-based tool to aid these operational platform employment decisions.

We introduce a new planning aid for operational-level mission planning of SPSS search. This planning aid will provide optimal placement and disposition of cooperating friendly search platforms while considering the intelligent response of the enemy SPSS operators to search efforts. A successful operation against an SPSS requires detection,

classification, and interdiction. The planning aid addresses detection and classification only, which are arguably the two most difficult stages of the operation.

We use a defender-attacker optimization model to maximize the defender's probability of successful detection and classification of the SPSS through the advantageous disposition of these search platforms against an intelligent attacker operating the semi-submersible. We assume the attacker has imperfect knowledge of defender platform disposition but is aware that there are defenders that must be avoided. Given this assumption, the solution to the defender-attacker model is a mixed (i.e., probabilistic) strategy for the defender and a least-risk path for the attacker.

We demonstrate our defender-attacker model with both an Eastern Pacific and a Caribbean scenario using five representative search platform types whose detection and classification performance vary by platform, and by geography. In each of these cases, we find that our model prescribes a face-valid defensive plan; defenders take advantage of geography by positioning at chokepoints in constrained waterways, and they provide coverage near attacker origins and destinations in the less geographically-constrained scenarios.

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I. INTRODUCTION

This thesis develops an operational level mission-planning aid to counter Self Propelled Semi-Submersibles (SPSS) currently being used by drug smugglers in Colombia to move large volumes of cocaine toward the U.S. This planning aid will suggest optimal cooperative placement and disposition of friendly search platforms while considering the intelligent response of the enemy SPSS operators to our search efforts. A successful operation against an SPSS requires detection, classification, and interdiction. This thesis addresses detection and classification only, which are arguably the two most difficult stages of the operation.

The primary use of this research is determining resource versus risk to gauge the benefit of additional platforms and capabilities or the effect of eliminating platforms and capabilities. The goal is to discover the best mix of platforms to use against the SPSS as well as their optimal cooperative disposition in terms of placement and the spread of their search effort. Major inputs to the model are positions where the SPSS can enter the sea space and the goal positions were it can leave the sea space. A smaller number of these positions implies better friendly intelligence and will result in improved search platform performance. By increasing or decreasing the number of entry and goal positions, the value of intelligence can be captured.

A. SPSS THREAT

The SPSS is not a true submarine, but a surface vessel with very low freeboard (Figures 1 and 2). A typical SPSS has a hull that only rises a foot above the waterline. Maritime drug traffickers in the Caribbean and Eastern Pacific have historically used a combination of commercial fishing vessels and smaller, harder to detect “go fast” boats. The SPSS combines the low profile of a “go-fast” boat with the long range and high capacity of fishing boat. SPSSs have been in limited use for nine years, but they have become an increasingly popular choice for Columbian drug smugglers in the Eastern Pacific because efforts at policing fishing boats have improved in recent years (Bajak, 2008). Twenty-three SPSSs were launched from Colombia between 2006 and 2007

(Wilkenson, 2008). In 2008, nearly 70 were seen or captured (StrategyPage, 2009). SPSSs now convey up to 75% of the estimated 600 tons of cocaine originating from Colombia annually. The vessels have been operating nearly exclusively in the Pacific making the transit from Colombia to Mexico or Guatemala (Brown, 2009). SPSS technology is easily exportable to other locations. At least one has been used by a local drug gang in Spain (StrategyPage 2009).

Though SPSSs are currently being used exclusively for drug trafficking, they could easily be used to transport any manner of contraband. Both the Director of Joint Interagency Task Force (JIATF) South and Commander, United States Southern Command (COMUSSOUTHCOM) have suggested that SPSSs could be used to support terrorism (Brown, 2009). COMUSSOUTHCOM cited the delivery of weapons of mass destruction as a worst case scenario (USSOUTHCOM, 2008).

SPSS have small visual, infra-red and radar signatures making them difficult to detect by non-acoustic sensors. Recently SPSS designers have taken particular care to reduce infra-red signature through the use of exhaust cooling systems (Brown, 2009). Classification is also difficult for most sensors. Because each SPSS is purpose-built, and configurations change constantly, there is no telltale signature for an SPSS. They look similar to sailboats on radar and sound similar to fishing boats acoustically. As a result, an SPSS can easily blend into legitimate maritime traffic. These difficulties are compounded by a vast area of operation twice the size of the continental U.S. It is little wonder that only an estimated 10% of SPSS are interdicted (Wilkenson, 2008). Many of these successful interdictions are the result of intelligence information at the source (StrategyPage, 2009). Our odds of success can be improved either through improved intelligence or through improved maritime domain awareness. The focus of this study is on improving maritime domain awareness through the optimized employment of scarce search assets, but the value of intelligence can also be captured as a byproduct of the analysis.

Figure 1. Various SPSS Designs at Bahia Malaga Navy Base Colombia (From Bajak, 2008).

Figure 2. Captured SPSS at NAS Key West (From Brown, 2009).

B. COUNTERING THE SPSS THREAT

1. Search Platforms

There are several types of search platforms (i.e., units to detect, classify, and interdict SPSSs) being employed by the Joint Interagency Task Force (JIATF) South to combat the SPSS (e.g., Figures 3-5). These include ships such as partner nation frigates, U.S. Navy frigates and U.S. Coast Guard cutters. Aircraft can include helicopters embarked from these ships as well as land-based maritime patrol aircraft (MPA) and airborne early warning (AEW) aircraft operated by partner nations, U.S. Air Force, U.S. Navy, U.S. Coast Guard, and U.S. Customs and Border Patrol. Consideration is also being given to employing non-traditional acoustic sensors such as Seaweb (Figure 6), a proposed autonomous network of acoustic sensors that communicate with each other through acoustic modems (Rice, 2009). Submarines are considered given the SPSSs relatively loud acoustic signature (at least relative to what submarines traditionally hunt). Though a variety of platforms that can be used against the SPSS, there are generally only a few available at any given time.

Figure 3. U.S. Coast Guard C-130 Maritime Patrol Aircraft (From Wikipedia.org 2009).

Figure 4. U.S. Navy Frigate (From Wikipedia.org 2009).

Figure 5. U.S. Customs and Border Patrol P-3 Airborne Early Warning Aircraft (From Wikipedia.org 2009).

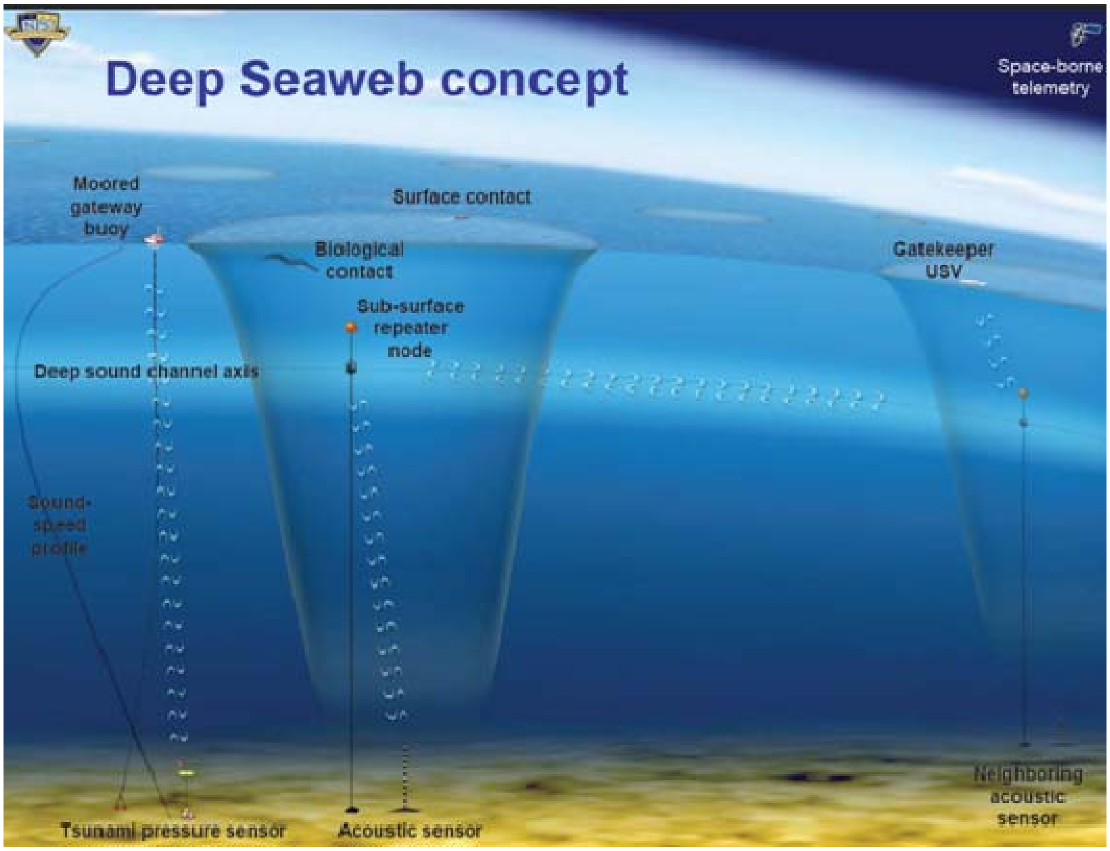

Figure 6. Seaweb is an Autonomous Networked Array of Acoustic Sensors (From Rice 2008).

2. Current Planning Methods

Because searching for a single SPSS can tie up many search assets current SPSS interdiction relies heavily on actionable prior intelligence. This greatly reduces the search area. In the absence of prior intelligence operational commanders rely on information such as cones of courses and speed versus time with consideration for background effects such as weather to help determine the most likely routes for the SPSS. Care is taken to not be predictable in employing search assets, since smugglers have demonstrated the ability to avoid known drug interdiction efforts. There are no computer-based tools to aid these operational platform employment decisions. There are tactical level computer based planning aids such as the Personal Computer-Based Interactive Multisensor Analysis Training System (PCIMAT) designed for anti-submarine warfare that can help predict the performance of acoustic sensors. However, these decision aids only predict the performance of individual sensors; they cannot coordinate the simultaneous, cooperative disposition of several platforms in an optimal manner against an intelligent adversary.

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II. MODEL DEVELOPMENT

A. NETWORK REPRESENTATION

We model the maritime environment using a mesh network (Figure 7). The ocean area is broken down into 60nm square cells with a node in the middle of each. Each cell node is connected by an arc to and from adjacent cells unless there is an obstruction to navigation, such as land. SPSSs typically begin their journey from small river estuaries or sparsely-populated coastline where our maritime platforms cannot effectively operate. Therefore, the entry cells are only navigable by the SPSS (hereafter called the "attacker"). SPSSs typically deliver their cargo offshore where maritime search platforms can operate. Therefore, goal cells are navigable by both the attacker and defender.

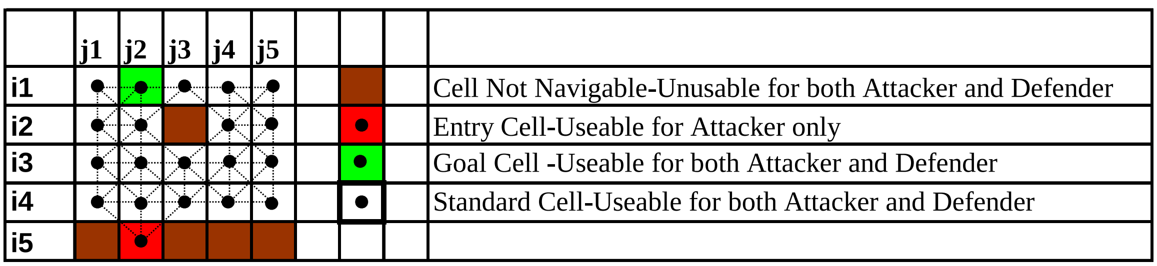


Figure 7. Example Network Representation. Each dot represents the center of a 60nm square cell of sea space, and each line connecting a pair of dots represents an adjacency indicating it is possible to navigate directly between these cells.

We assume the attacker behaves intelligently, and maximizes his probability of evading detection and classification by choosing a directed path from an advantageous entry cell node to a an advantageous goal cell node that has the maximum joint probability of evading detection and classification along all arcs in the path. If we assume independence between arcs, the joint probability that the attacker will evade detection and classification is the product of the evasion probabilities over each of the arcs the attacker transits. Maximizing the sum of the logarithms of the evasion probabilities is equivalent to maximizing the product of these probabilities because the logarithm is a single-valued increasing function (e.g., Abdul-Ghaffar, 2008).

B. MISSION GENERATION

The first step in determining the arc-based log evasion probabilities is the generation of all possible defender missions and their pre-calculated cell-based log evasion probabilities. A mission has two parameters, the set of cells searched, $c \in C_{pm} \subseteq C$ and the *type* of search platform $p \in TYPE \subseteq P$ (i.e. MPA, Seaweb, frigate etc). We call the cardinally of the set of cells searched the *size* of the mission. Appendix A lists the types of search platforms, their detection capabilities, classification capabilities and the maximum size of mission each type can perform. The geographic models in Appendix B describe how detection and classification capabilities vary by cell. The defender may have several platforms of the same type (e.g. two frigates and three Marine Patrol Aircraft) but each platform is only capable of performing the missions $m \in M_p \subseteq M$ generated for that type of platform.

We begin generating missions by building all the possible shapes for the set of cells in the mission for each mission size. We do not attach cells in a mission diagonally since the attacker can pass through such an attachment without penalty. We allow rectangle shapes of all sizes, “L” shapes of size three, and stair-step shapes of even sizes (Figures 8-10).

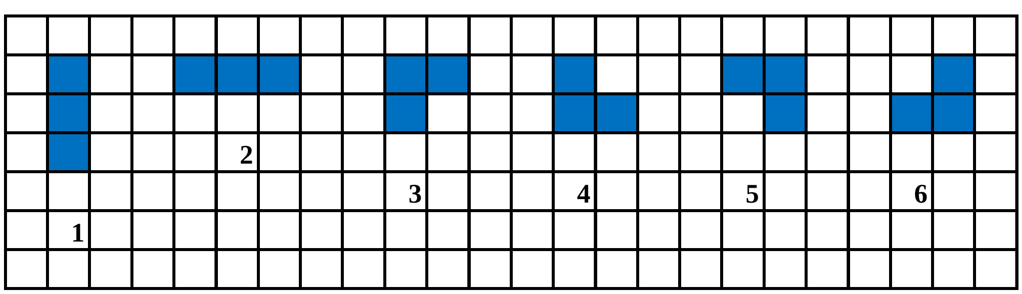


Figure 8. Generated mission shapes of size three.

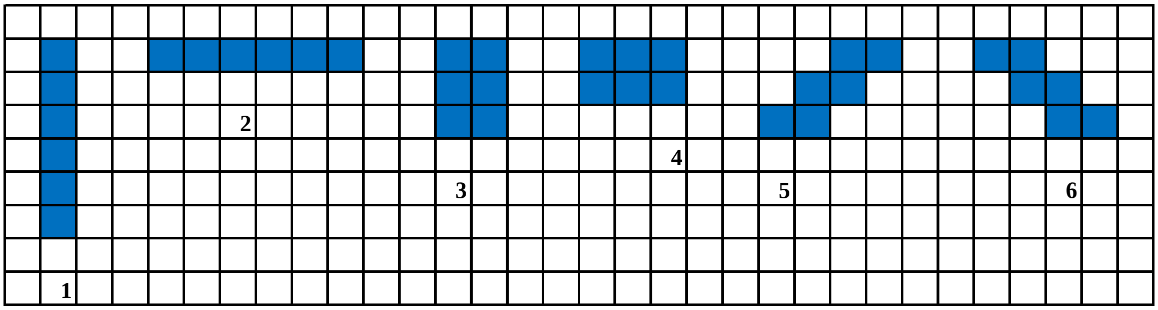


Figure 9. Generated Mission Shapes of Size Six.

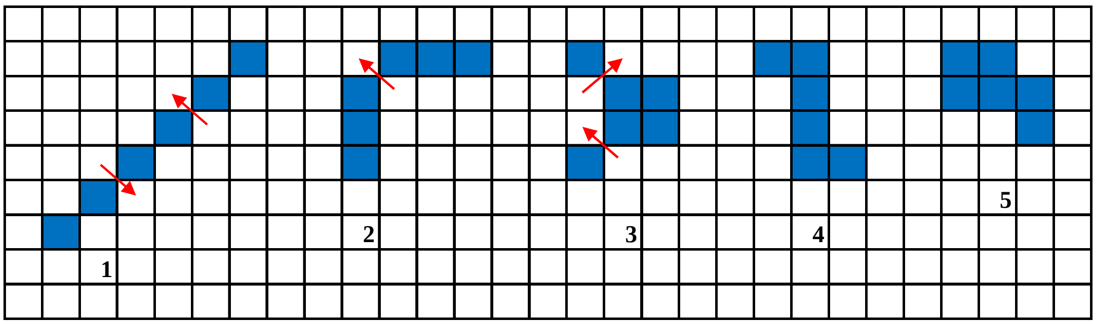


Figure 10. Some Restricted Mission Shapes of Size Six. Arrows Depict Zero Penalty Paths.

Once we have generated all the shapes of each size, we generate all the sets of search platform navigable cells that can be searched by each shape. These sets of cells cannot include non-navigable cells or entry cells. We then combine these sets of cells with our platform types to create our candidate missions. As discussed in Appendix A, some searcher types are able to perform missions of larger size than others. For example, a frigate is capable of searching missions up to size three, while a MPA can search up to size ten.

We calculate the log evasion probability for every cell in a mission using both detection probability and the classification probability $class_{p,c}$. The data and assumptions used to determine classification probability are detailed in Appendices A and B. We use the random search formula to calculate the probability of detection. This results in a conservative estimate of the probability of detection. With w being the sensor sweep width, v being speed and t being the time the attacker can transit a cell, and A being the area cell being searched, then by the random search formula (Wagner 1999 p.174):

$$P_d = 1 - e^{-wvt/A}.$$

For moving search platforms such as MPA we use the searcher speed for v. For fixed platforms such as Seaweb we use the attacker's speed. We call wvt/A the coverage factor $cf_{p,m,c}$. The calculation of coverage factor for every platform type is detailed in Appendix A. Additional data used in the calculation of coverage factor is contained in Appendix B. Because the log evasion probability is lower when the attacker transits diagonally than rectilinearly (horizontally or vertically), we calculate separate diagonal and rectilinear and log evasion probabilities.

<u>Indices and index sets [~cardinality]</u>

$c \in C$	cells with horizontal, vertical coordinates (alias c1, c2) [~500]
$m \in M$	possible missions [~50,000]
$p \in P$	defending platforms [~10]
$p \in TYPE \subseteq P$	type of defending platform [5]
$m \in M_p \subseteq M$	missions platform p can perform [~15,000]
$c \in C_{pm} \subseteq C$	cells patrolled by platform p while performing mission m [~20]

<u>Data [units]</u>

$cf_{p,m,c}$	coverage factor of platform p, performing mission m, in cell c (See Appendices A and B) [nondimensional].
$class_{p,c}$	classification probability given detection of platform p in cell c (See Appendices A and B) [probability]
$size_m$	number of cells in mission m [1-20]

Calculated Data [units]

$ev\_rec_{p,m,c}$ log likelihood that an attacker traversing cell *c* rectilinearly would evade detection and classification by defender *p* performing mission *m* patrolling cell *c* [log likelihood]

$$ev\_rec_{p,m,c} = \log[(1-(1-e^{-cf_{p,m,c}})class_{p,c}] \quad \forall p \in P,\ \forall c \in C_{p,m}, \forall m \in M_p$$

$ev\_diag_{p,m,c}$ log likelihood that an attacker traversing cell *c* diagonally would evade detection and classification by defender *p* performing mission *m* patrolling cell *c* [log likelihood]

$$ev\_diag_{p,m,c} = \log[(1-(1-e^{-\sqrt{2}cf_{p,m,c}})class_{p,c}] \quad \forall p \in P,\ \forall c \in C_{p,m}, \forall m \in M_p$$

Once we have these log evasion probabilities, we can calculate the log probability that the attacker traversing a cell will evade detection and classification given an incumbent defender solution. We need to account for cases where the attacker traverses a cell rectilinearly and diagonally.

New Data [units]

$X_{p,m}$ Incumbent solution. 1 if platform *p* performs mission *m*, 0 otherwise [binary]

New Calculated Data

ev_recX_c log of probability that an attacker will evade detection and classification traversing cell *c* rectilinearly [log likelihood]

$$ev\_recX_c = \sum_{p \in P, m \in M_p} ev\_rec_{p,m,c} X_{p,m} \qquad \forall c \in C$$

ev_diagX_c log of probability that an attacker will evade detection classification traversing cell *c* rectilinearly [log likelihood]

$$ev\_diagX_c = \sum_{p \in P, m \in M_p} ev\_diag_{p,m,c} X_{p,m} \qquad \forall c \in C$$

C. CELL DATA TO ARC DATA CONVERSION

We now have cell-based log evasion probabilities for every cell in the network. We need to covert these to arc-based log evasion probabilities. Figure 11 depicts an example path from cell (i2, j1) to cell (i1, j3). As the attacker uses arc [(i2, j1), (i1, j2)], he is subject to the diagonal log evasion of cell (i2, j1) for half the arc and the diagonal log evasion of cell (i1, j2) for half the arc. Similarly, as the attacker uses arc [(i1, j2), (i1, j3)] he is subject to the rectilinear log evasion of cell (i2, j1) for half the arc and the rectilinear log evasion of cell (i1, j3) for half the arc. If we assume independence, we can add ½ the log evasion of the tail cell to ½ the log evasion of the head cell to calculate the total log evasion for the arc.

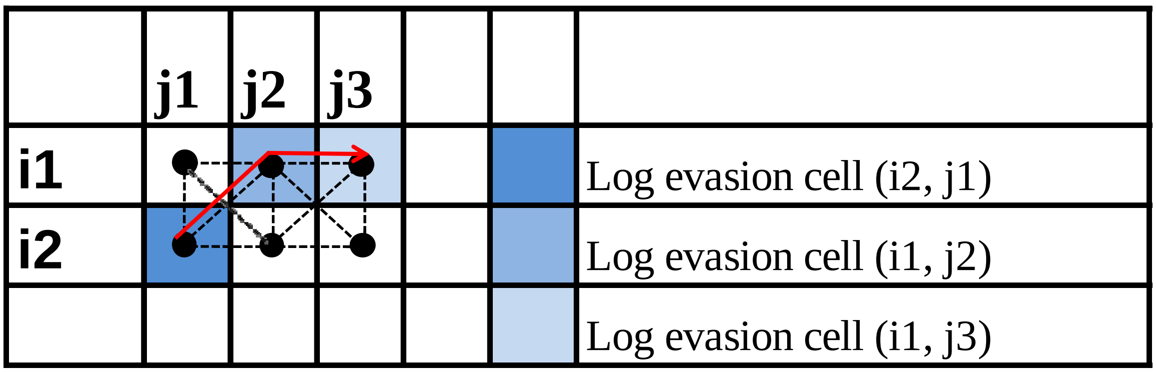


Figure 11. Example half arc log evasion calculation. The total log evasion for the path equals ½ the diagonal log evasion for cell (i2, j1) plus ½ the diagonal log evasion for cell (i1, j2) plus ½ the rectilinear log evasion for cell (i1, j2) plus ½ the rectilinear log evasion for cell (i1, j3).

<u>New indices and index sets [~cardinality]</u>

$d \in D_{c1,c2} = D$ cell adjacencies, or traversal arcs [~4,000]

$d \in D_{rec}$ rectilinear arcs

$d \in D_{diag}$ diagonal arcs

<u>New Calculated Data [units]</u>

evX_d log of probability that an attacker will evade detection and classification traversing arc *d* [log likelihood]

$$evX_d = \frac{1}{2}(ev\_recX_{c1} + ev\_recX_{c2}) \quad \forall d \in D_{rec}$$

$$evX_d = \frac{1}{2}(ev\_diagX_{c1} + ev\_diagX_{c2}) \quad \forall d \in D_{diag}$$

$ev_{d,p,m}$ log likelihood that an attacker traversing arc *d* would evade detection and classification by defender *p* performing mission *m* [log likelihood]

$$ev_{d,p,m} = \frac{1}{2}(ev\_rec_{p,m,c1} + ev\_rec_{p,m,c2}) \quad \forall d \in D_{rec}, m \in M_p$$

$$ev_{d,p,m} = \frac{1}{2}(ev\_diag_{p,m,c1} + ev\_diag_{p,m,c2}) \quad \forall d \in D_{diag}, m \in M_p$$

The log evasion on an arc is more negative when the probability of detection and classification is high. On arcs where there are no defenders present the log evasion is at its maximum value of zero (i.e., evasion probability one). Thus, the attacker incurs no penalty for traversing arcs without a defender present. This allows the attacker in an evasion-maximizing model to take arbitrary and unrealistic meandering paths, cycles, and sub-tours. The defender will attempt to interdict these meandering paths with an unrealistic sensor platform laydown rather than the more reasonable and realistic paths between an entry cell and a goal cell. To prevent the attacker from taking meandering paths we add a small penalty to every arc called *fuel*. If *fuel* is kept large, it induces no unreasonable behavior on the part of the attacker. In probability, this represents a small background chance that an attacker will be detected by complete accident, even though the detection is not made in a cell with an assigned search mission. In the absence of a defender the attacker will take the shortest path to a goal cell, and the attacker will take a longer path to a goal cell in order to avoid a defender.

<u>New Data [units]</u>

fuel fuel penalty for traversing a single arc [log likelihood of evasion]

<u>Updated Calculation</u>

$$evX_d = fuel + \frac{1}{2}(ev\_recX_{c1} + ev\_recX_{c2}) \quad \forall d \in D_{rec}$$

$$evX_d = fuel + \frac{1}{2}(ev\_diagX_{c1} + ev\_diagX_{c2}) \quad \forall d \in D_{diag}$$

D. MODEL FORMULATION

Once we have both the cell-based and arc-based log evasion data, we use a modification of the formulation used by Ghaffar in his planning to optimize sensor placement against terrorist "go-fast" boats (Abdul-Ghaffar, 2008, pp. 14-19).

1. The Attacker

The attacker has a single SPSS that can choose to enter a network at any one of a number of entry cells $c \in E$, traverse a set of cell-to-cell arcs $d \in D$ to reach (and exit the network at) any one of a number of goal cells $c \in G$. Traversing each arc carries a risk of detection and classification the attacker cannot control, and the log likelihood that an attacker will evade detection and classification while traversing arc *d* is evX_d. The attacker seeks paths that maximize the log likelihood of evading detection.

We express the attackers' planning problem with the model **SSV_SP(ev**X**)**:

<u>New Indexes and index sets [~cardinality]</u>

$c \in E \subseteq C$ cells where an attacker can enter the network [~10]

$c \in G \subseteq C$ goal cells [~10]

<u>Variables [units]</u>

$ENTER_c$ 1 if the attacker enters network at entry cell *c*, 0 otherwise [binary]

Y_d 1 if the attacker traverses arc *d*, 0 otherwise [binary]

$GOAL_c$ 1 if the attacker exits network at goal cell *c*, 0 otherwise [binary]

<u>Formulation [dual variables]</u>

$$Z_{\max}(\widehat{evX}) = \max_{Y} \sum_{d \in D} \widehat{evX}_{d} Y_d \qquad \text{(A0)}$$

$$s.t. \quad \sum_{c \in E} ENTER_c \leq 1 \qquad [\alpha] \qquad \text{(A1)}$$

$$\sum_{d \in D_{c,c2}} Y_d - \sum_{d \in D_{c1,c}} Y_d$$
$$- ENTER_c\big|_{c \in E} + GOAL_c\big|_{c \in G} \leq 0 \quad \forall c \in C \qquad [\beta_c] \qquad \text{(A2)}$$

$$-\sum_{c \in G} GOAL_c \leq -1 \qquad [\delta] \qquad \text{(A3)}$$

Discussion

The attackers' objective (A0) is to maximize the total expected log likelihood that attackers traversing arcs from entry cells on paths to goal cells evade detection and classification (or, equivalently, to maximize the joint probability that they evade detection and classification over all the arcs they choose to traverse). Constraint (A1) limits the entry into the network to the entry cells, each constraint (A2) forces conservation of flow at a cell in the network, and constraint (A3) limits the exit from the network to goal cells. This linear program will produce an intrinsically integral solution Y^* (e.g., Ahuja et al. 1993, pp. 447-449). (The Greek notation associated with each constraint denotes a linear programming dual variable used later.)

2. The Defender

The defender controls a set of search platforms (e.g., MPA, frigates, etc.) $p \in P$ that may each be performing a mission $m \in M_p$. The log likelihood that an attacker traversing arc d evades detection and classification by defender platform p performing mission m is $ev_{d,p,m}$. The defender seeks positions for his search platforms to minimize the total log likelihood of attackers evading his surveillance. We express the defender's problem as follows **DMIN($\hat{\mathbf{Y}}$)**

New indices and index sets [~cardinality]

$m \in M$ possible missions [~50,000]

$p \in P$ defending platforms [~10]

$m \in M_p \subseteq M$ missions platform p can perform [~10,000]

New data [units]

$ev_{d,p,m}$ log likelihood that an attacker traversing arc d would evade detection and classification by defender p performing mission m [log likelihood]

$\hat{Y}_d$ 1 if attacker traverses arc d, 0 otherwise [binary]

Variables [units]

$X_{p,m}$ =1 if platform p performing mission m in cell c, 0 otherwise [binary]

Z total log likelihood of evading detection and classification [log likelihood]

Formulation

$$Z_{\min}(\hat{Y}) = \min_{X,Z} Z \qquad \text{(D0)}$$

$$s.t. \quad Z \geq \sum_{\substack{d \in D, p \in P \\ m \in M_p}} ev_{d,p,m} \hat{Y}_d X_{p,m} \qquad \text{(D1)}$$

$$s.t. \quad \sum_{m \in M_p} X_{p,m} \leq 1 \qquad \forall p \in P \qquad \text{(D2)}$$

$$\sum_{p \in P} X_{p,m} \leq 1 \qquad \forall m \in M_p \qquad \text{(D3)}$$

$$\sum_{(p,m): m \in M_p \wedge c \in C_{pm}} X_{p,m} \leq 1 \qquad \forall c \in C \qquad \text{(D4)}$$

$$X_{p,m} \in \{0,1\} \qquad \forall p \in P,\ \forall m \in M_p \qquad \text{(D5)}$$

Discussion

(D0) introduces the objective, and constraint (D1) defines the objective variable as the minimum upper bound on total log likelihood of evasion. Each constraint (D2) allows a defender platform to perform at most one mission, each constraint (D3) allows a mission to be performed by at most one platform, each constraint (D4) allows at most one platform occupy a cell, and (D5) stipulates a binary decision for each platform mission.

3. Defender-Attacker (D-A) Model

We initially assume that attacker has perfect knowledge of these mission choices and responds by choosing a path that maximizes the cumulative log evasion probability created by these mission choices. These sequential actions by the defender and the attacker are a type of Stackelberg game represented as a defender-attacker (D-A) model (e.g., Brown et al., 2006). The following initial formulation **SPSS-MINMAX** is a min-max formulation that cannot be solved through conventional means.

$$Z = \min_{Z,X} \max_{Y} \sum_{\substack{d \in D, \\ p \in P, m \in M_p}} ev_{d,p,m} X_{p,m} Y_d$$

$$s.t. \qquad \text{(A1)} - \text{(A3) and } \left(\text{D1}\right) - \text{(D4)}$$

4. Dual Integer Linear Program Formulation

A standard technique in solving D-A is to temporarily fix *Z* and *X*, take the dual of the remaining problem (**SSV_SP(**$\mathbf{ev}X$**)**), and then release *Z* and *X* to obtain a single integer linear program that can be solved conventionally (e.g., Brown et al., 2006). We express this integer linear program as **SPSS-ILP.**

$$\min_{\substack{\alpha,\beta,\delta, \\ X}} \quad \alpha - \delta \qquad \text{(T0)}$$

$$\begin{array}{lllr}
 & \alpha - \beta_c \geq 0 & \forall c \in E & \text{(T1)} \\
s.t. & -\beta_{c1} + \beta_{c2} \geq \sum_{\substack{p \in P, \\ c \in C_{pm}}} ev_{d,p,m} X_{p,m} & \forall d \in D_{c1,c2} & \text{(T2)} \\
 & -\delta + \beta_c \geq 0 & \forall c \in G & \text{(T3)} \\
 & \sum_{m \in M_p} X_{p,m} \leq 1 & \forall p \in P & \text{(T4)} \\
 & \sum_{(p,m): m \in M_p \wedge c \in C_{pm}} X_{p,m} \leq 1 & \forall c \in C & \text{(T5)} \\
 & \alpha \geq 0 & & \\
 & \beta_c \geq 0 & \forall c \in C & \\
 & \delta \geq 0 & & \\
 & X_{p,m} \in \{0,1\} & \forall p \in P, m \in M_p & \text{(T6)}
\end{array}$$

SPSS-ILP solves the D-A problem by choosing platform missions *X*, and then recovers optimal attacker paths by fixing *X* and solving **SSV_SP(ev**X**)**.

5. Decomposition

The **SPSS-ILP** is difficult to solve on a large scale. A Benders decomposition applies naturally to D-A problems. The standard Benders method takes the dual of **SPSS-ILP** with *X* fixed which leads back to **SPSS-MINMAX** (e.g., Brown et al. 2006). We modify **DMIN(Ŷ)** replacing equation (D1) with (D1K). This is the Bender's decomposition **Master Problem**. **SSV_SP(ev**X**)** is the Bender's decomposition **Subproblem.**

<u>New index</u>

$k \in K$ decomposition iteration

<u>New Data</u>

$\hat{Y}_k$ attacker plans for iteration *k*

DMIN(Ŷ) <u>Modification</u>

$$Z_{\min}(\hat{Y}) = \min_{Z,X} Z$$

$$s.t. \qquad Z \geq \sum_{\substack{d \in D, \\ p \in P, m \in M_p}} ev_{d,p,m} \hat{Y}_d^k X_{p,m}, \quad k=1,\ldots,K \quad \text{(D1K)}$$

Constraints (D2)-(D4) unchanged.

The complete decomposition algorithm is as follows:

Algorithm SPSS Decomposition

Input: Data for defense problem, optimality tolerance $\varepsilon \geq 0$;

Output: ε-optimal defender location plan $\mathbf{X}^*$, and responding attacker plan $\mathbf{Y}^*$;

1. Initialize best upper bound $Z_{UB} \leftarrow \infty$, best lower bound $Z_{LB} \leftarrow -\infty$, define the incumbent, null defender plan $\mathbf{X}^* \leftarrow \hat{\mathbf{X}}^1 \leftarrow \mathbf{0}$ as the best found so far, and set iteration counter $K \leftarrow 1$;

2. **Subproblem**: Calculate **ev**$\mathbf{X}_d$ using $\hat{\mathbf{X}}^K$. Solve subproblem **SSV_SP(ev**X**)** to determine the optimal attack plan $\hat{\mathbf{Y}}^K$ given $\hat{\mathbf{X}}^K$; the bound on the associated objective is $\overline{Z}_{\max}(\hat{\mathbf{X}}^K)$;

3. If ($Z_{UB} > \overline{Z}_{\max}(\hat{\mathbf{X}}^K)$) set $Z_{UB} \leftarrow \overline{Z}_{\max}(\hat{\mathbf{X}}^K)$ and record improved incumbent defender plan $\mathbf{X}^* \leftarrow \hat{\mathbf{X}}^K$, and responding attacker plan $\mathbf{Y}^* \leftarrow \hat{\mathbf{Y}}^K$;

4. If ($Z_{UB} - Z_{LB} \le \varepsilon$) go to **End**;

5. **Master Problem:** Given attack plans $\hat{\mathbf{Y}}^k$, k=1,…K, attempt to solve master problem **DMIN($\hat{\mathbf{Y}}$)** to determine an optimal defender plan $\hat{\mathbf{X}}^{K+1}$. The bound on the objective is $\underline{Z}_{\min}(\hat{\mathbf{Y}})$;

6. If $Z_{LB} < \underline{Z}_{\min}(\hat{\mathbf{Y}})$ set $Z_{LB} \leftarrow \underline{Z}_{\min}(\hat{\mathbf{Y}})$;

7. If ($Z_{UB} - Z_{LB} \le \varepsilon$) go to **End**;

8. Set $K \leftarrow K + 1$ and go to step (2) (**Subproblem)**;

9. **End**: $\mathbf{X}^*$ is an ε-optimal defender solution, and $\mathbf{Y}^*$ is the attacker response to that plan.

6. Binary Relaxation

We initially assume that the defender's platforms are operating overtly and the attacker has perfect knowledge of ("observes") defender actions.

The assumption of perfect attacker knowledge is a conservative assumption, and perhaps not a very realistic one in light of the low-technology nature of an SPSS. In the covert case, we assume that the attacker knows the number and type of platforms that may be searching but does not know their exact disposition. In this case, the optimization becomes a simultaneous two-person zero sum game (Von Neumann et al., 2004). The attacker may need to randomize path(s) and the defender may need to randomize mission assignments.

The formulations of the overt case and covert case are nearly identical. In order to formulate the simultaneous-play game in the covert case the binary restriction of X and Y are relaxed to continuous restrictions (Brown et al., 2008, p. 97). The resulting X is the probability that the defender chooses to employ a platform in a mission. We refer to this probability as a mixed strategy, though it may turn out in practice to by a pure one (i.e., all probability devoted to just one mission).

E. SOFTWARE IMPLEMENTATION

We implement the SPSS decomposition algorithm using General Algebraic Modeling System (GAMS) (GAMS, 2009) with ILOG CPLEX (ILOG, 2007) set as the linear programming solver. We use Visual Basic for Applications (VBA) to support this GAMS code. We calculate $ev\_rec_{p,m,c}$ and $ev\_diag_{p,m,c}$ in VBA using the assumptions from Appendix A with the geographic models from Appendix B contained in Microsoft Excel worksheets. Additionally, VBA uses the GAMS output to generate the probability field and mixed strategy figures discussed in the next chapter.

III. RESULTS AND ANALYSIS

Typical solutions of the SPSS decomposition algorithm take 5 hours to complete 270 iterations using a windows based personal computer with a Xeon X5460 processor. A typical final iteration instance of $\mathbf{SSV\_SP}(\widehat{\mathbf{ev}}X)$ has 3,000 variables and 450 constraints with 10,000 non-zero elements. A typical final iteration instance of $\mathbf{DMIN}(\hat{\mathbf{Y}})$ has 280,000 variables and 700 constraints with 4.1 million non-zero elements.

For our scenarios, we use five different representative search platform types. The characteristics of these platform types are summarized in Table 1 and detailed in Appendix A. The coverage factor $cf_{p,m,c}$ used in the calculation of the log evasion is proportional to speed of the platform (or speed of the SPSS in the case of Seaweb) and sweep width. The sweep width of each sensor type varies by geographic factors summarized in Table 1 and detailed in Appendix B. For each platform type, the classification probability $class_{p,c}$ used the calculation of log evasion varies by the density of other shipping activity in each cell.

Platform type	**max size**	**Classification Probability**	**Speed (knots)**	**Sweep width(nm)**	**Sweep width depends on**
MPA	10	0.95-0.80	180	0-17.4	wind speed
AEW	10	0.5-0.l	180	0-43.5	wind speed
Frigate	3	0.95-0.80	10	10-30	wind speed, water depth, shipping density
Submarine	3	0.5-0.l	10	10-30	wind speed, water depth, shipping density
Seaweb	1 (per sensor)	0.5-0.1	6 (SPSS speed)	0-37 (per sensor)	water depth

Table 1. Sensor platform characteristics. For instance, a frigate can cover missions searching up to 3 cells, with classification probability 0.95-0.80, with speed 10 knots and sweep width 10-30 nautical miles, depending on factors including wind speed, water depth, and shipping density in the searched cells.

From Table 1 we see that AEW aircraft are better at detection than MPA but the MPA is better at classification. Given its wider sweep width the AEW aircraft's detection probability degrades less than the MPA's detection probability as the mission size is increased to its maximum of 10 cells. We therefore expect the defender to choose missions of larger size for AEW aircraft than for MPA. MPA's classification performance does degrade with shipping density by not to the same degree as AEW. We therefore expect the defender to use MPA in areas of high shipping density compared to AEW. Both of these platforms are best used in areas where the wind speed is low. The detection performance of both the frigate and the submarine are hampered by their slow speed. The detection performance of these platforms degrades quickly as the size of the mission increases. We expect most selected frigate and submarine missions to be of size one. Though both platforms are affected by acoustic conditions (wind, water depth, and shipping density), the submarine's classification probability is affected more acutely by shipping density than the frigate. We expect the defender to avoid using the submarine in areas of high shipping density. Water depth has the most influence on Seaweb detection performance. In deep water, a ten-sensor Seaweb network has similar detection performance to an MPA. Like the submarine and AEW aircraft, Seaweb is relatively poor classifier and is better used in areas of low shipping density.

We restrict the total mixed strategy probabilities of all platform assignments to any given cell be less than or equal to one. Sending a platform to a given cell with too high a probability precludes the use of that cell for other platforms. A relatively poor sensor platform, such as the submarine in a desirable cell, might keep that cell from being used by more capable platforms. These less capable platforms can be useful in confined areas, such as chokepoints. It might be better to employ a frigate in a chokepoint and allow an MPA to spread its effort of over more cells in less confined areas.

A. CARIBBEAN BASELINE SCENARIO

We choose the Caribbean as the setting for our first scenario (Figure 12). The geography of the Caribbean is more conducive to model validation than the Pacific. There are several chokepoints that the SPSS must transit to reach any goal. Given a reasonable number of entry and goal cells, a valid solution should make use of these chokepoints. The Caribbean is also a plausible location for export of SPSS technology. Though SPSS have operated nearly exclusively in the Eastern Pacific, the Caribbean has been used as a transit zone for other vehicles used in drug smuggling, such as go-fast boats and aircraft. For this baseline scenario, we use one each of MPA, AEW, frigate and submarine platform types. We assume that we have one of these assets on station at all times. In addition, we employ two separate Seaweb networks with 10 sensors each.

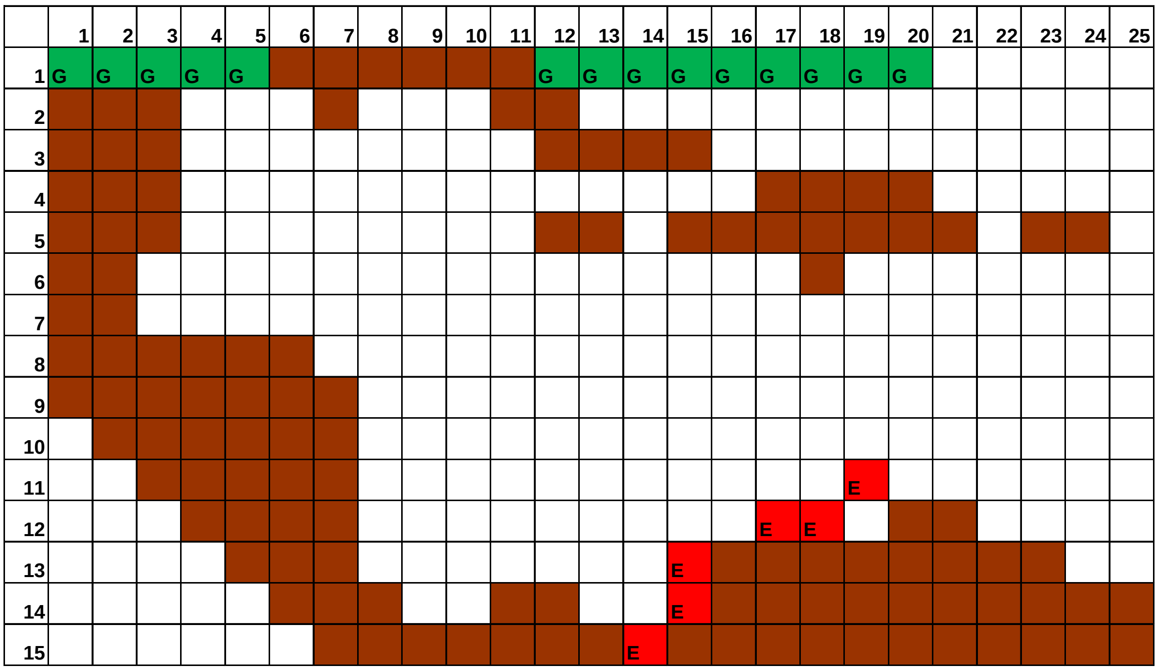


Figure 12. Caribbean Baseline Scenario Entry and Goal Cells. Each cell is 60 nautical miles square. This depiction shows latitudes 08-23N and longitudes 64-88W. On the west is the Mexican and Central American coast, on the south is Columbia, where each “E” cell is a possible entry, and to the north is the Gulf coast of the U.S., where each “G” cell is a possible goal. Islands present obstructions to navigation for both attacker and defender.

1. Caribbean Mixed Strategy

The mixed strategy we select is the probability the defender should adopt each mission for any given time epoch. The number in each cell indicates the probability a search platform is sent to a cell. The higher probability cells are darkly shaded, and the lower probability cells are lightly shaded. Figures 13-17 depict individual platform mixed strategies.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
1																				0.017	0.017	0.011			
2				0.198	0.128	0.162														0.011	0.017	0.017			
3				0.214		0.162	0.162									0.056	0.021			0.011	0.011	0.036	0.006		
4				0.214			0.07	0.07						0.328	0.383	0.056					0.021	0.04	0.006	0.006	
5				0.214										0.347								0.135			0.067
6				0.016										0.02	0.02						0.095	0.095		0.067	0.067
7										0.003															
8										0.003															
9										0.003															
10										0.003															
11																									
12																									
13																									
14																									
15																									

Figure 13. Caribbean MPA Mixed Strategy. Cells with dark outlines are entry and goal cells. Numeric values are search platform-to-cell probabilities of assignment. The shading distinguishes qualitatively between relatively high search probabilities and low ones. We have chosen a probabilistic barricade, utilizing available geographic chokepoints to navigation. Suggested missions are generally of size three or four cells.

MPA is used primarily to cover the chokepoints. When compared with the shipping density model in Appendix B, we see that MPA are sent to areas of high shipping density. We also see from the wind model in Appendix B that MPA avoid any areas where the wind is so high that the sweep width is zero. The defender tends to choose MPA missions of size three or four.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
1																									
2								0.002	0.002							0.026	0.026	0.026	0.026	0.026					0.016
3					0.434	0.441	0.547	0.117	0.004							0.026	0.026	0.026	0.026	0.026					0.016
4				0.009	0.443	0.473	0.547	0.086	0.004																0.016
5				0.058	0.536	0.491	0.51	0.082	0.007																0.016
6				0.039	0.097	0.067	0.095	0.094	0.014	0.017	0.029	0.072	0.072	0.091	0.091	0.053					0.012	0.012	0.012	0.062	0.078
7						0.058	0.16	0.122	0.043	0.041	0.06	0.07	0.079	0.075	0.075	0.056					0.012	0.012	0.012	0.062	0.078
8							0.085	0.128	0.079	0.126	0.088	0.069	0.069	0.054	9E-04	0.01	0.009							0.049	0.066
9								0.124	0.123	0.134	0.11	0.107	0.066	0.053			0.009	0.009						0.049	0.066
10								0.066	0.057	0.078	0.052	0.044	0.013					0.009	0.009					0.049	0.049
11								0.008	0.005	0.005	0.013	0.025	0.018												
12								0.008	0.005	0.005	0.005	0.025	0.025												
13												0.012	0.019	0.007											
14																									
15																									

Figure 14. Caribbean AEW Mixed Strategy. Here we see much wider-ranging mission assignments.

AEW aircraft tend to spread in areas of open ocean but still use these open ocean areas to cover the chokepoints. Most suggested missions are of size nine or ten. Checking the shipping density and wind models in Appendix B reveals that the defender AEW aircraft avoid areas with high shipping density and high winds with a few exceptions that merely allow assignment of attractive larger mission sizes.

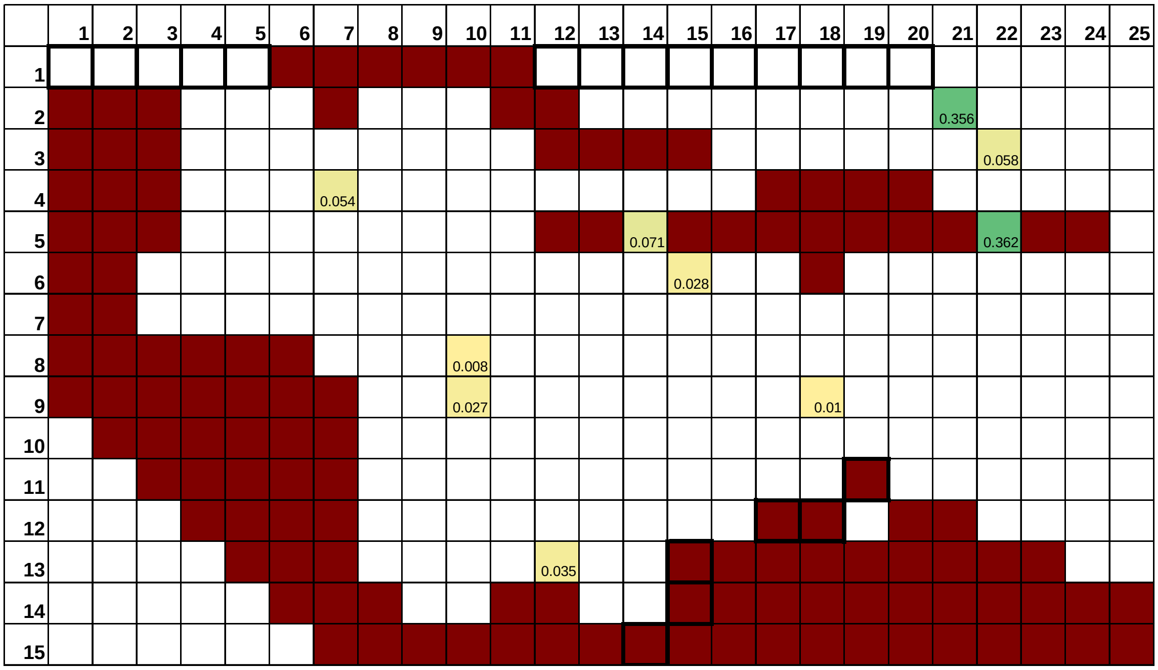


Figure 15. Caribbean Frigate Mixed Strategy.

The frigate is used in a few constrained areas, but there is also a tendency to avoid areas that may be better used by other platforms. When comparing to the shipping density and acoustic condition models in Appendix B, we see that all of the cells chosen are of moderate or high shipping density and all have moderate to good acoustic conditions. All but one of the frigate missions suggested for the defender are of size one.

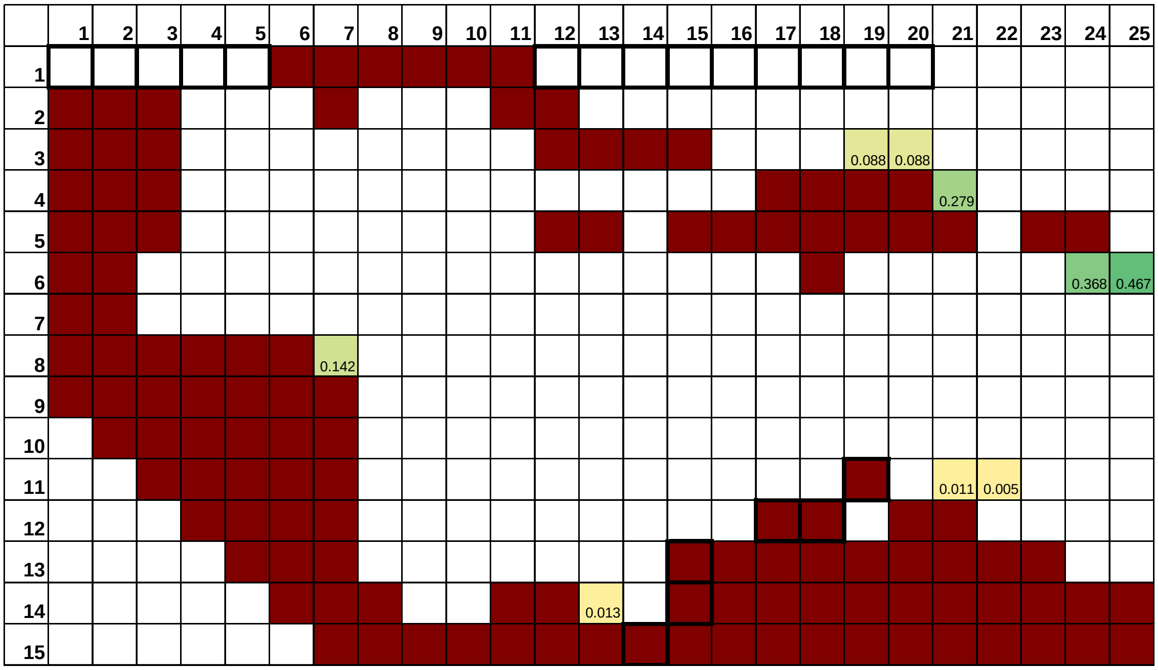


Figure 16. Caribbean Submarine Mixed Strategy.

When comparing to the shipping density and acoustic condition models in Appendix B, we see that all of the cells chosen are of low shipping density and all have moderate to good acoustic conditions. The most likely missions chosen for the submarine are of size one.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
1																					0.709	0.506	0.069	0.01	
2																			0.073	0.073	0.627	0.438	0.041	0.014	
3					0.031	0.292	0.292	0.031												0.073	0.7	0.411	0.027	0.041	0.014
4							0.261														0.7	0.484			
5																									
6															0.004	0.004				0.002		0.097	0.348	0.504	0.388
7																0.004	0.004	0.012	0.012	0.013		0.124	0.259	0.384	0.504
8													0.029	0.029	0.067	0.039	0.052	0.063	0.035	0.033	0.024	0.032	0.043		
9												0.031	0.14	0.168	0.249	0.25	0.242	0.242	0.233	0.227	0.219	0.17	0.154	0.067	
10										8E-04	8E-04	0.024	0.106	0.106	0.071	0.043				0.002	0.005	0.032	0.043		
11											8E-04	0.051	0.05	0.04	0.016								0.012		
12											0.026	0.051	0.041	0.04											
13																									
14																									
15																									

Figure 17. Caribbean Seaweb Mixed Strategy.

Checking the depth model in Appendix B reveals advantageous placement of Seaweb in deep water to cover the entry cells as well as the eastern goal cells. The chokepoints are too shallow for Seaweb to be used effectively. Examining the shipping density model in Appendix B reveals that we suggest placing Seaweb in areas of high shipping density only when the water is deep and the cell is part of a larger mission in a desirable area for search. The size of the missions chosen by the defender vary by water depth. The larger mission sizes are in deeper water.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
1																				0.017	0.726	0.518	0.069	0.01	
2				0.198	0.128	0.162		0.002	0.002							0.026	0.026	0.026	0.099	0.11	1	0.455	0.041	0.014	0.016
3				0.214	0.465	0.895	1	0.148	0.004							0.082	0.047	0.026	0.115	0.199	0.711	0.504	0.033	0.041	0.03
4				0.223	0.443	0.473	0.931	0.155	0.004					0.328	0.383	0.056					1	0.523	0.006	0.006	0.016
5				0.272	0.536	0.491	0.51	0.082	0.007					0.419								0.497			0.084
6				0.055	0.097	0.067	0.095	0.094	0.014	0.017	0.029	0.072	0.072	0.111	0.143	0.057				0.002	0.107	0.205	0.36	1	1
7						0.058	0.16	0.122	0.043	0.043	0.06	0.07	0.079	0.075	0.075	0.06	0.004	0.012	0.012	0.013	0.012	0.136	0.271	0.446	0.582
8							0.227	0.128	0.079	0.137	0.088	0.069	0.098	0.083	0.068	0.049	0.061	0.063	0.035	0.033	0.024	0.032	0.043	0.049	0.066
9								0.124	0.123	0.163	0.11	0.137	0.206	0.221	0.249	0.25	0.251	0.262	0.233	0.227	0.219	0.17	0.154	0.116	0.066
10								0.066	0.057	0.081	0.052	0.068	0.119	0.106	0.071	0.043		0.009	0.009	0.002	0.005	0.032	0.043	0.049	0.049
11								0.008	0.005	0.005	0.013	0.076	0.068	0.04	0.016						0.011	0.005	0.012		
12								0.008	0.005	0.005	0.032	0.076	0.066	0.04											
13												0.046	0.019	0.007											
14													0.013												
15																									

Figure 18. Overall Mixed Strategy. This reminds us that the prior, platform-by-platform mixed strategies are, in fact, an orchestrated, simultaneous, coordinated, joint employment plan for all of them. Over time this shows how we suggest all the platforms be assigned.

Figure 18 depicts the overall mixed strategy. This shows where the defender chooses to send all assets. A cell is saturated if the total mixed strategy probability for all defenders equals 1.0. We see the defender saturates the easternmost chokepoint with search platforms.

2. Caribbean Probability Field

We combine the mixed strategy with sensor performance to create the probability field in Figure 19. More darkly shaded cells have a high probability of detection and classification that the attacker wishes to avoid. The numbers in the probability field cells are based on a rectilinear transit. The probabilities based on a diagonal transit are higher but the relative strength of these probabilities from cell to cell is similar for rectilinear and diagonal transits.

The path depicted by the arrow is the worst-case attacker path from the defender's point of view. The probability of detection and classification given this path is approximately 0.69. Any other path taken by the attacker results in a higher probability of detection and classification. Of course, the attacker cannot always take this worst-case path. If the attacker always used this worst case path, the defender would be able to defend against this one path rather than all possible paths.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
1																				0.009	0.222	0.148	0.032	0.005	
2				0.111	0.1	0.135		8E-04	8E-04							0.007	0.013	0.013	0.037	0.043	0.23	0.123	0.013	0.005	0.009
3				0.188	0.232	0.311	0.491	0.067	0.002							0.061	0.04	0.013	0.029	0.038	0.254	0.09	0.017	0.016	0.014
4				0.188	0.206	0.219	0.261	0.078	9E-04					0.445	0.251	0.05					0.253	0.098	0.002	0.003	0.009
5				0.285	0.243	0.226	0.231	0.04	0.004					0.393								0.315			0.077
6				0.042	0.048	0.033	0.046	0.045	0.007	0.008	0.009	0.023	0.035	0.076	0.072	0.027				3E-04	0.136	0.173	0.091	0.3	0.271
7						0.011	0.047	0.032	0.011	0.014	0.033	0.034	0.038	0.036	0.036	0.015	8E-04	0.002	0.002	0.002	0.006	0.033	0.073	0.14	0.183
8							0.09	0.062	0.039	0.039	0.042	0.033	0.038	0.031	0.012	0.011	0.007	0.011	0.003	0.006	0.004	0.007	0.011	0.024	0.033
9								0.06	0.059	0.053	0.053	0.056	0.056	0.054	0.044	0.043	0.041	0.033	0.04	0.04	0.041	0.035	0.033	0.035	0.033
10								0.032	0.029	0.024	0.017	0.025	0.024	0.018	0.013	0.007		0.003	0.002	4E-04	6E-04	0.007	0.011	0.024	0.024
11								0.004	0.001	0.003	0.003	0.019	0.013	0.006	0.002						0.002	6E-04	0.002		
12								0.005	0.001	0.003	0.003	0.019	0.01	0.003											
13												0.027	0.009	0.002											
14													0.003												
15																									

Figure 19. Caribbean Probability Field and Worst-Case Attacker Path.

B. PACIFIC BASELINE SCENARIO

The Pacific is a more realistic setting than the Caribbean in that drug cartels currently use large numbers SPSSs here. There are no chokepoints, but there are fewer entry cells than goals cells (Figure 20). Unless there is a marked difference in environmental conditions between the areas bordering entry and goal cells, we expect the defender to concentrate his assets to cover the area surrounding entry cells. As in the Caribbean, the defender has one MPA, one AEW aircraft, one frigate, one submarine, and two Seaweb networks with ten sensors each. Figures 21-26 depict the Pacific mixed strategy.

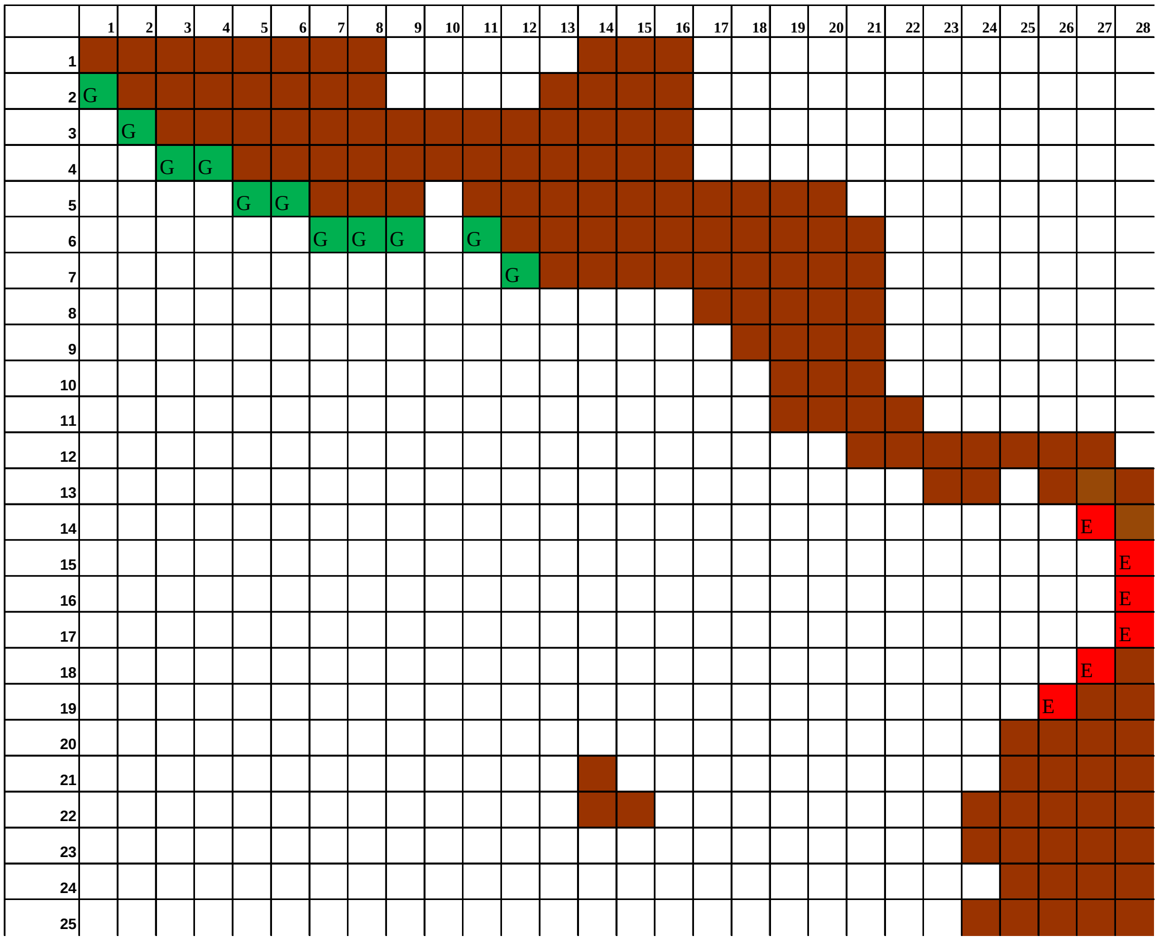


Figure 20. Pacific Baseline Scenario Entry and Goal Cells. Each cell is a 60 nautical mile square. At the lower right (south-east) is the coast of Columbia with entry cells “E”, while at the upper left (north-west) is the coast of Mexico with goal cells “G”.

1. Pacific Mixed Strategy

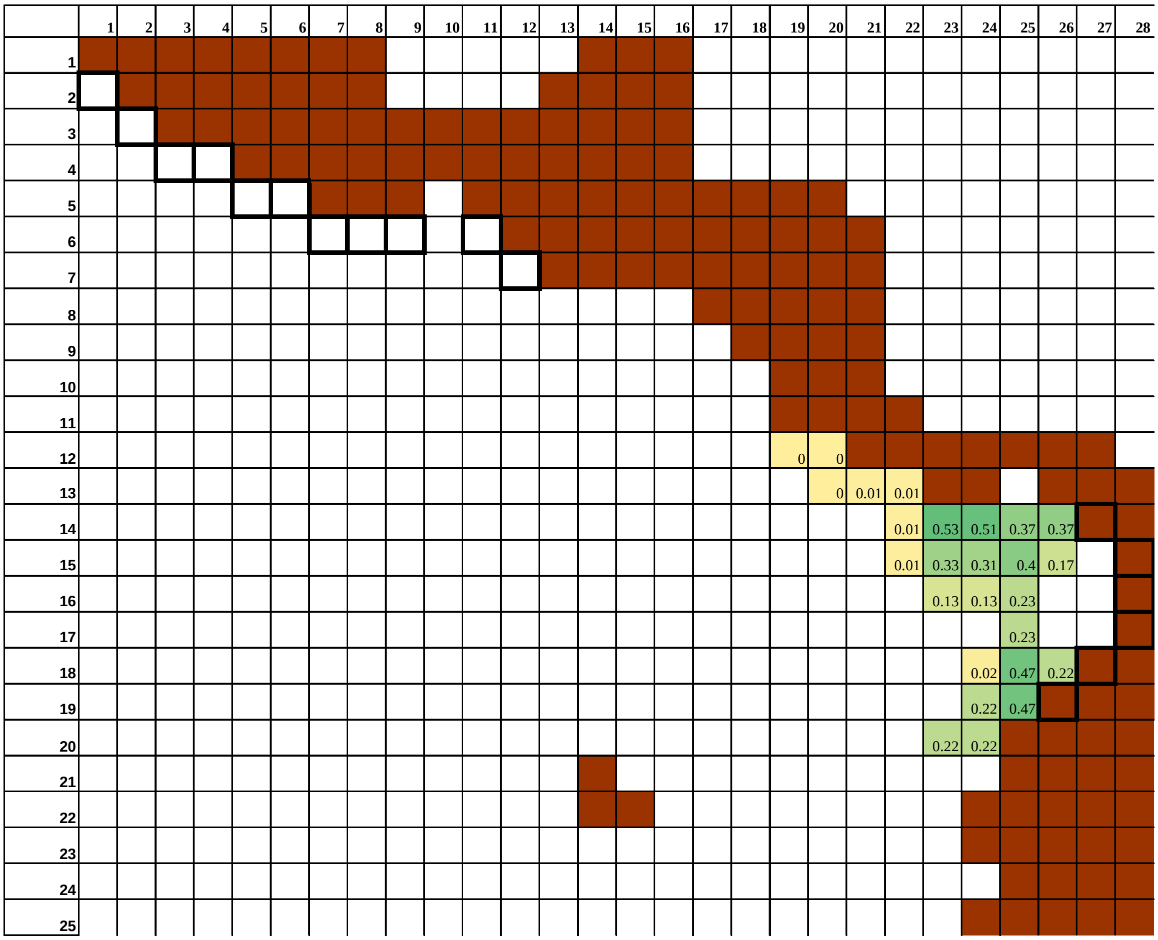


Figure 21. Pacific MPA Mixed Strategy. We see that in this case the suggested probabilistic barrier covers the entry cells.

MPA is used primarily to cover the entry points. When compared to the shipping density model in Appendix B, we see that MPA cover nearly all the high shipping density cells in the vicinity of the entry cells. We also see from the wind model in Appendix B that MPA avoids any areas where the expected wind is 20 knots or higher. The only high shipping density cell in the vicinity of the entry cells not covered by MPA has 20-knot winds. Due to lower winds and wider sweep widths, we preferentially choose MPA missions of larger size in the Pacific.

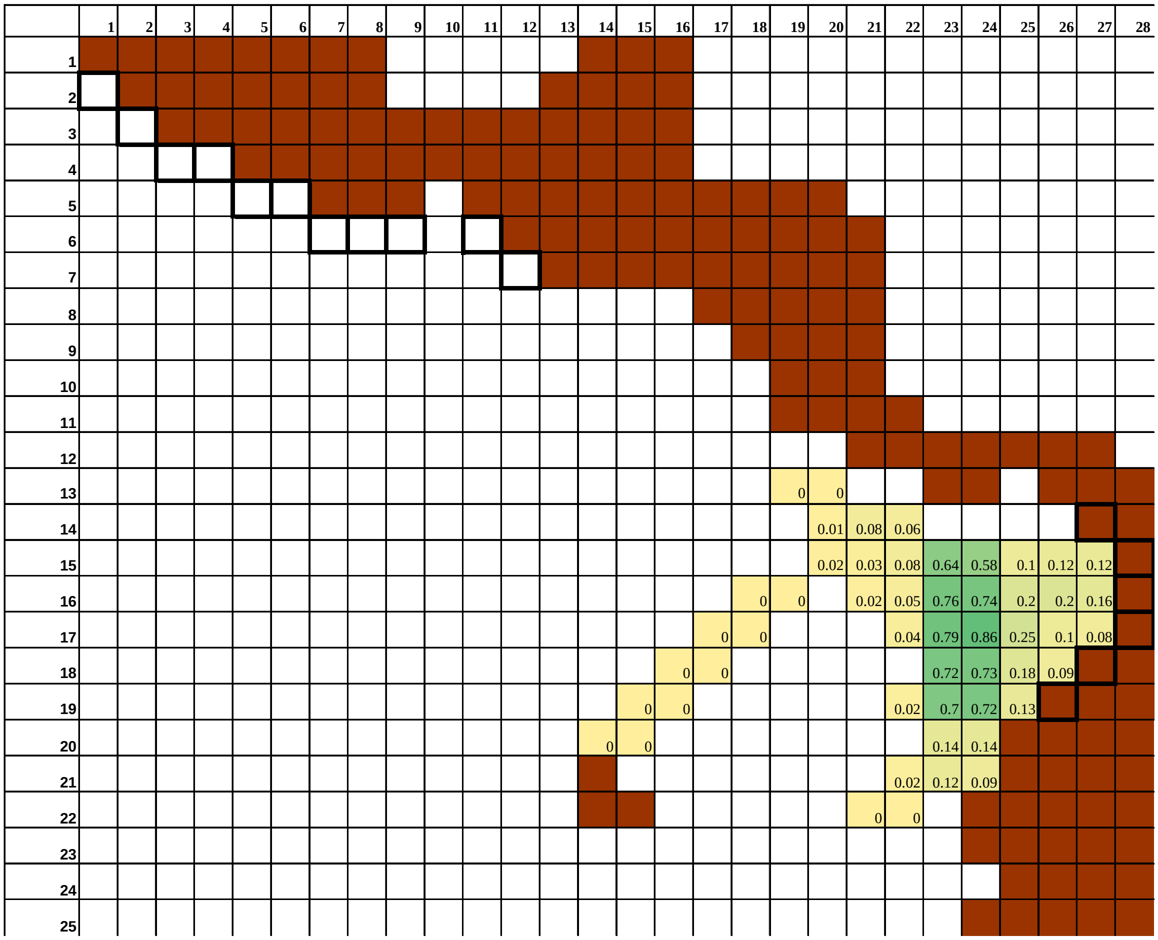


Figure 22. Pacific AEW Mixed Strategy. Similar to MPA, AEW aircraft concentrate their search effort near the entry cells.

As with MPA, AEW aircraft are better used to cover the entry points. As in the Caribbean, we prefer mission sizes of nine or ten. Checking the shipping density and wind models in Appendix B reveals that the defender AEW aircraft avoid areas with high shipping density and high winds with exception of i15, j25. Choosing this cell allows AEW defender to fit a mission with larger mission size within a relatively constrained navigable area.

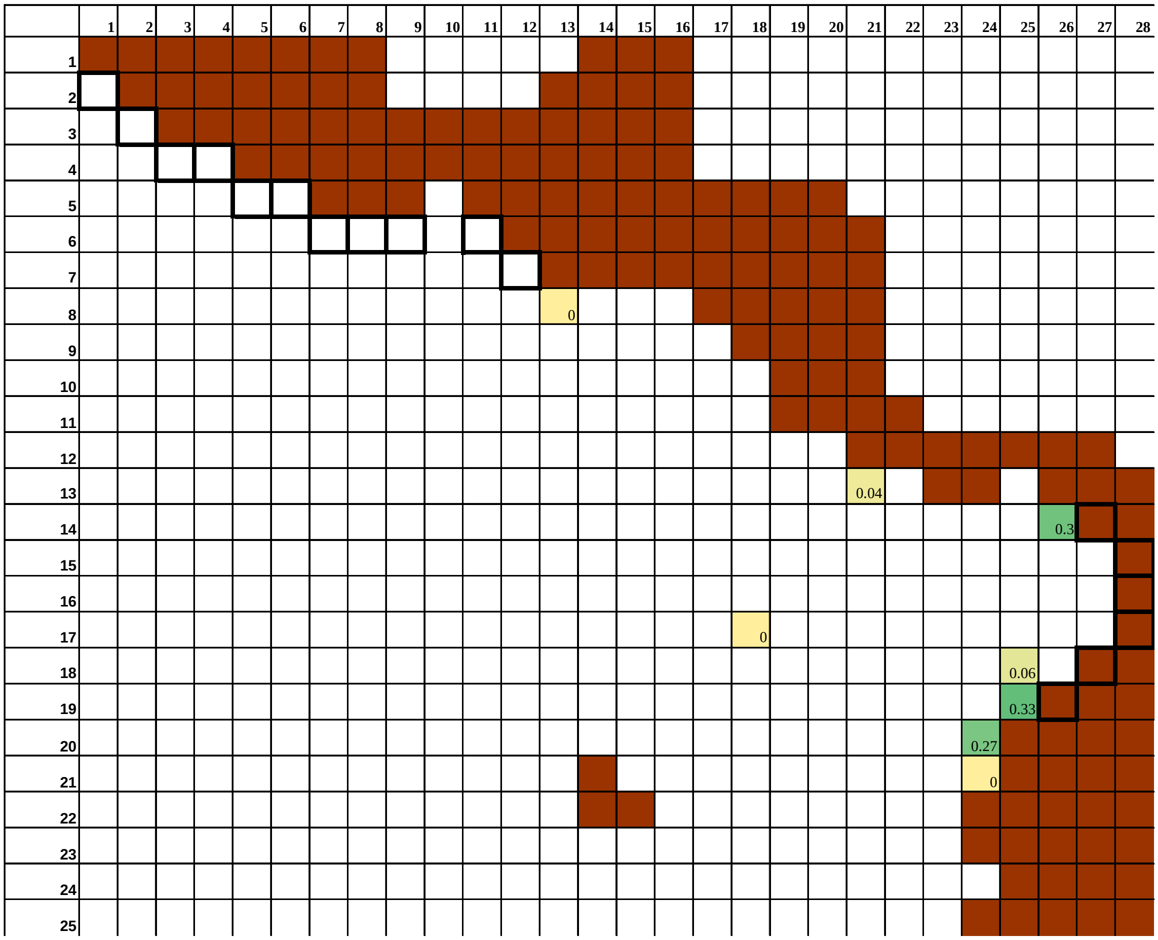


Figure 23. Pacific Frigate Mixed Strategy.

The frigate avoids areas that may be better used by other platforms and preferentially searches relatively constrained areas. From the acoustic model in Appendix B we see that the cells chosen by the defender have good acoustic conditions. Every frigate mission we suggest is of size one.

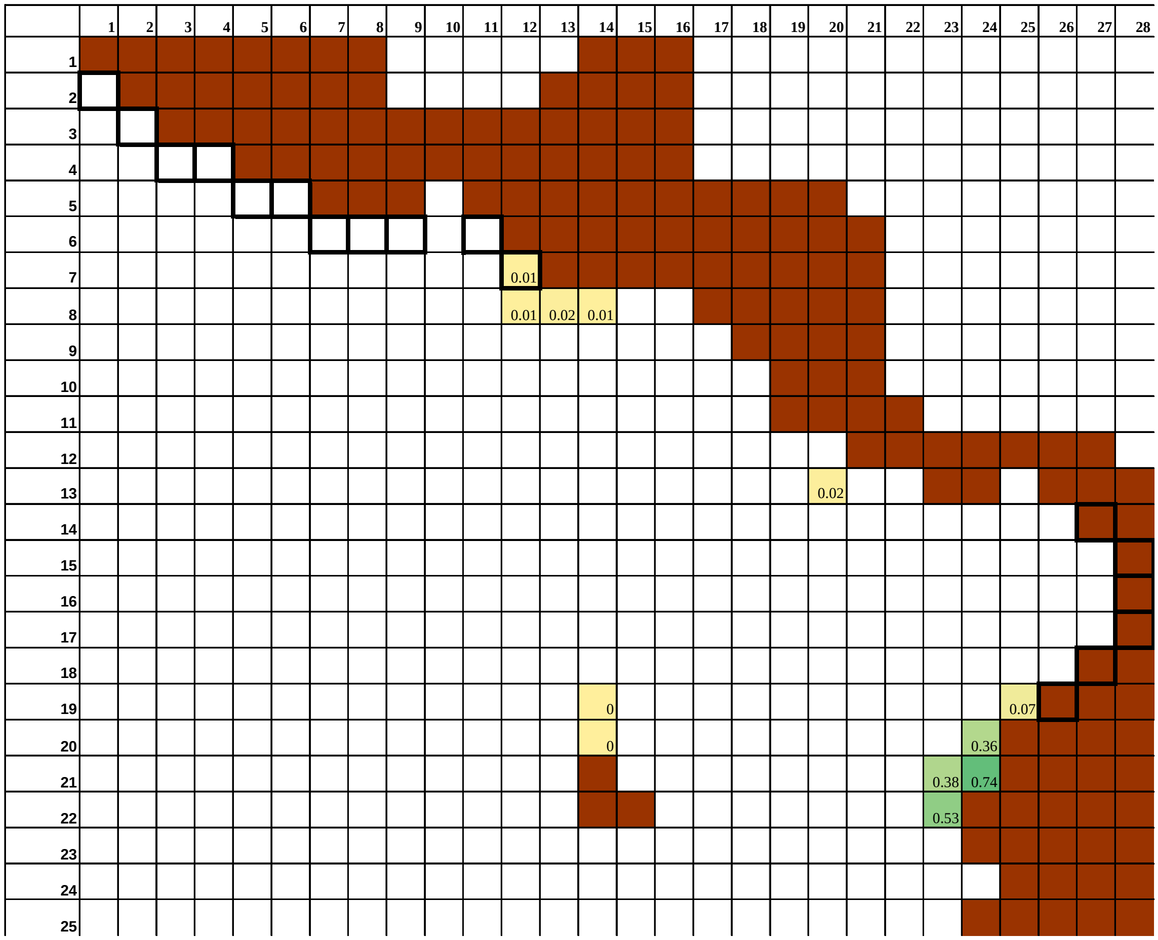


Figure 24. Pacific Submarine Mixed Strategy.

Checking the shipping density and acoustic condition models in Appendix B reveals that all of the cells suggested are of low to moderate shipping density and have moderate to good acoustic conditions. The submarine missions chosen are of size one or two.

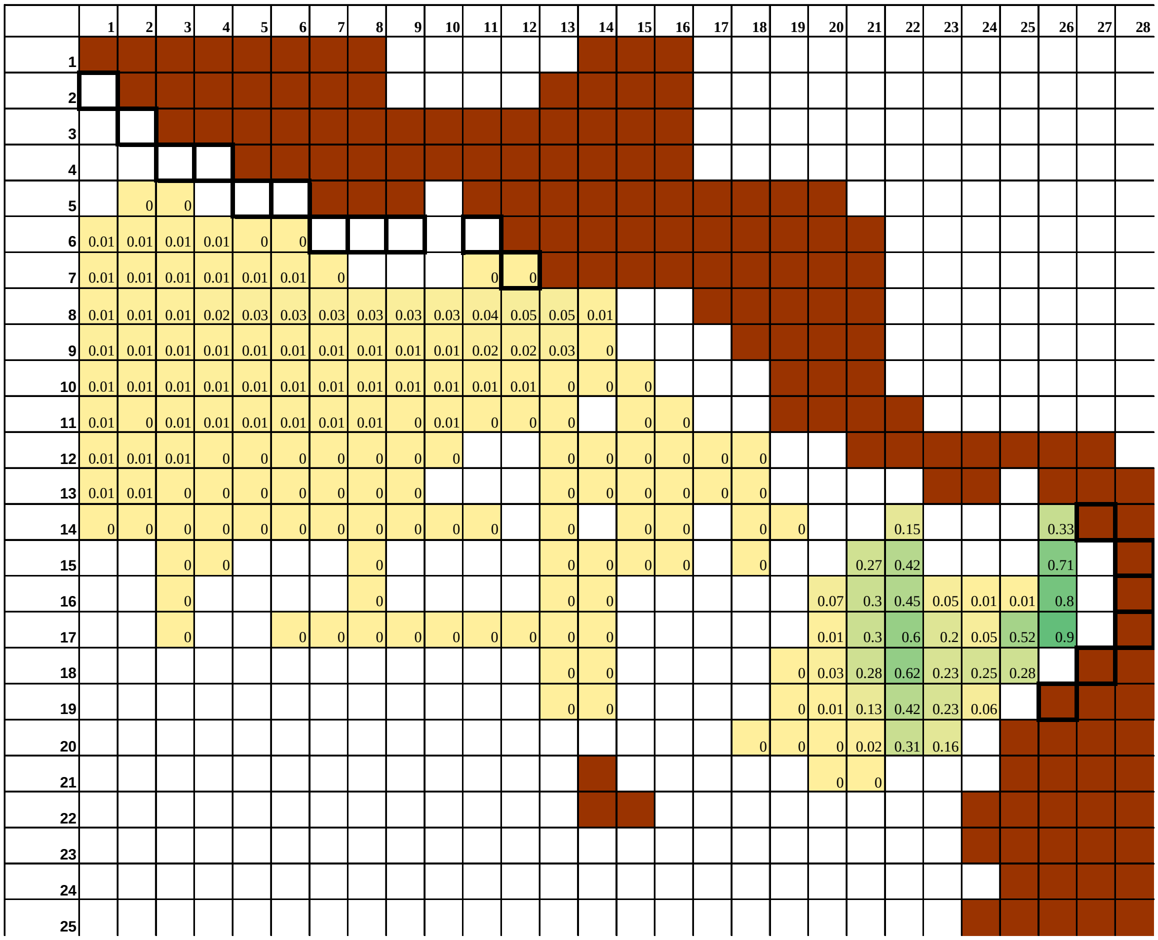


Figure 25. Pacific Seaweb Mixed Strategy. The overall, blended operational plan concentrates search effort near the entry cells. The shaded cells with "0" probability have non-zero probabilities too small to display.

Checking the depth model in Appendix B reveals we select a mixed strategy for Seaweb that closely matches the depth contours. Unlike the other sensor platforms, we devote some Seaweb effort to cover areas in the vicinity of goal cells. These areas are in deep water. Comparing with the shipping density model in Appendix B shows we only choose to use Seaweb in areas of moderate to low shipping density. The size of the missions suggested vary by water depth. The larger mission sizes are in deep water.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
1																												
2																												
3																												
4																												
5		0	0																									
6	0.01	0.01	0.01	0.01	0	0																						
7	0.01	0.01	0.01	0.01	0.01	0.01	0				0	0.01																
8	0.01	0.01	0.01	0.02	0.03	0.03	0.03	0.03	0.03	0.03	0.04	0.05	0.07	0.02														
9	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.02	0.02	0.03	0														
10	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0	0	0													
11	0.01	0	0.01	0.01	0.01	0.01	0.01	0.01	0	0.01	0	0	0		0	0												
12	0.01	0.01	0.01	0	0	0	0	0	0	0			0	0	0	0	0	0	0	0								
13	0.01	0.01	0	0	0	0	0	0	0				0	0	0	0	0	0	0	0.02	0.05	0.01						
14	0	0	0	0	0	0	0	0	0	0	0		0		0	0		0	0	0.01	0.08	0.23	0.53	0.51	0.37	1		
15			0	0				0					0	0	0	0		0		0.02	0.31	0.51	0.97	0.89	0.5	1	0.12	
16			0					0					0	0				0	0	0.07	0.32	0.5	0.94	0.88	0.44	1	0.16	
17			0			0	0	0	0	0	0	0	0	0			0	0		0.01	0.3	0.64	0.99	0.91	1	1	0.08	
18													0	0		0	0		0	0.03	0.28	0.62	0.95	1	1	0.31		
19													0	0	0	0			0	0.01	0.13	0.44	0.93	1	1			
20														0	0			0	0	0	0.02	0.31	0.52	1				
21																				0	0	0.02	0.5	0.84				
22																					0	0	0.53					
23																												
24																												
25																												

Figure 26. Pacific Overall Mixed Strategy. We suggest a mixed strategy that is a blockade of entry cells in Columbia.

We see in the overall mixed strategy that we suggest a probabilistic blockade in the immediate vicinity of entry cells.

2. Pacific Probability Field

We combine the mixed strategy of Figures 21-26 with sensor performance calculations to create the probability field in Figure 27. The path depicted by the arrow is the worst-case attacker path from the defender's point of view. The probability of detection and classification given this path is approximately 0.86. This probability is higher than that we achieve in the Caribbean due to a more favorable sensor environment.

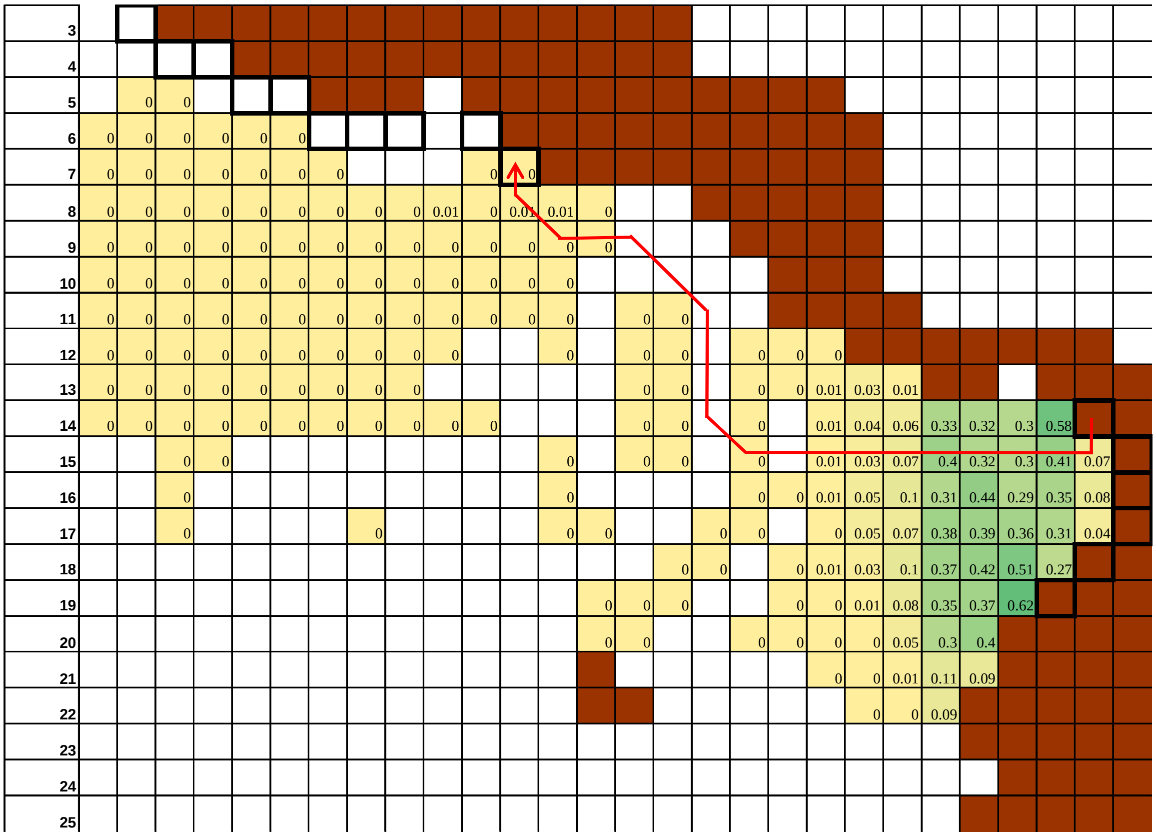


Figure 27. Pacific Probability Field and Worst-Case Attacker Path.

C. RESOURCES VERSUS RISK

We now explore the effect of removing MPA on the Baseline mixed strategy in the Caribbean (Figures 28-32). With the removal of MPA the defender adjusts the mixed strategy by using the AEW aircraft and Seaweb to cover the open ocean area between the entry cells and the two western chokepoints. These areas have low to moderate shipping densities that allow AEW aircraft and Seaweb to operate effectively (see Appendix B for shipping density and depth models). The westernmost chokepoint with high shipping densities now has no direct coverage. Similarly the defender deploys Seaweb to cover the eastern approaches to the goal cells by deploying in deep water in columns j21 and j22. The frigate strategy complements Seaweb in these columns by covering high shipping density cells.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
1																									
2								0.01								0.01	0.01	0.01	0.01	0.01					6E-04
3					0.002	0.011	0.011	0.011	0.002							0.01	0.01	0.01	0.01	0.01					6E-04
4					0.002	0.002	0.012	0.022	0.003	0.001	0.001	0.001	0.001	0.001	0.001	0.001									6E-04
5				0.008	0.008	0.005	0.005	0.019	0.009	1E-04	1E-04														6E-04
6					0.008	0.008	0.027	0.059	0.054	0.072	0.106	0.206	0.206	0.288	0.288	0.163								0.057	0.058
7						0.008	0.054	0.081	0.074	0.118	0.159	0.204	0.243	0.243	0.228	0.185	0.008	0.008						0.057	0.058
8							0.054	0.247	0.235	0.377	0.275	0.223	0.157	0.119		0.044	0.052	0.008						0.057	0.058
9								0.349	0.364	0.481	0.369	0.35	0.126	0.099			0.057	0.057						0.057	0.058
10								0.275	0.265	0.35	0.223	0.147	0.029			0.006	0.014	0.052	0.044					0.057	0.058
11								0.043	0.034	0.035	0.03	0.056	0.028		0.006	0.006	0.008	0.008							6E-04
12								0.004	0.004	0.005	0.005	0.056	0.053	0.006	0.006										
13												0.028	0.058	0.031											
14																									
15																									

Figure 28. AEW Mixed Strategy for Caribbean Scenario with no MPA.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
1																									
2																					0.321				
3																0.013						0.101			
4														0.03											
5														0.16								0.063			
6															0.179										
7																									
8										0.065															
9										0.1								0.007							
10																									
11																									
12																									
13												0.059													
14																									
15																									

Figure 29. Frigate Mixed Strategy for Caribbean Scenario with no MPA.

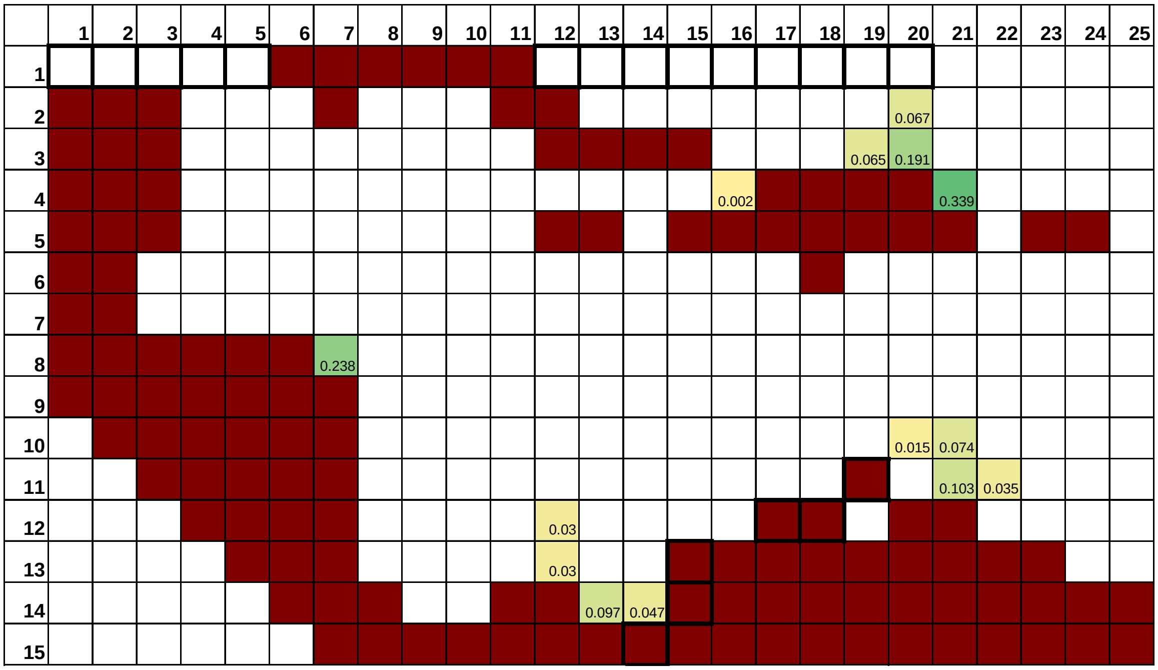


Figure 30. Submarine Mixed Strategy for Caribbean Scenario with no MPA.

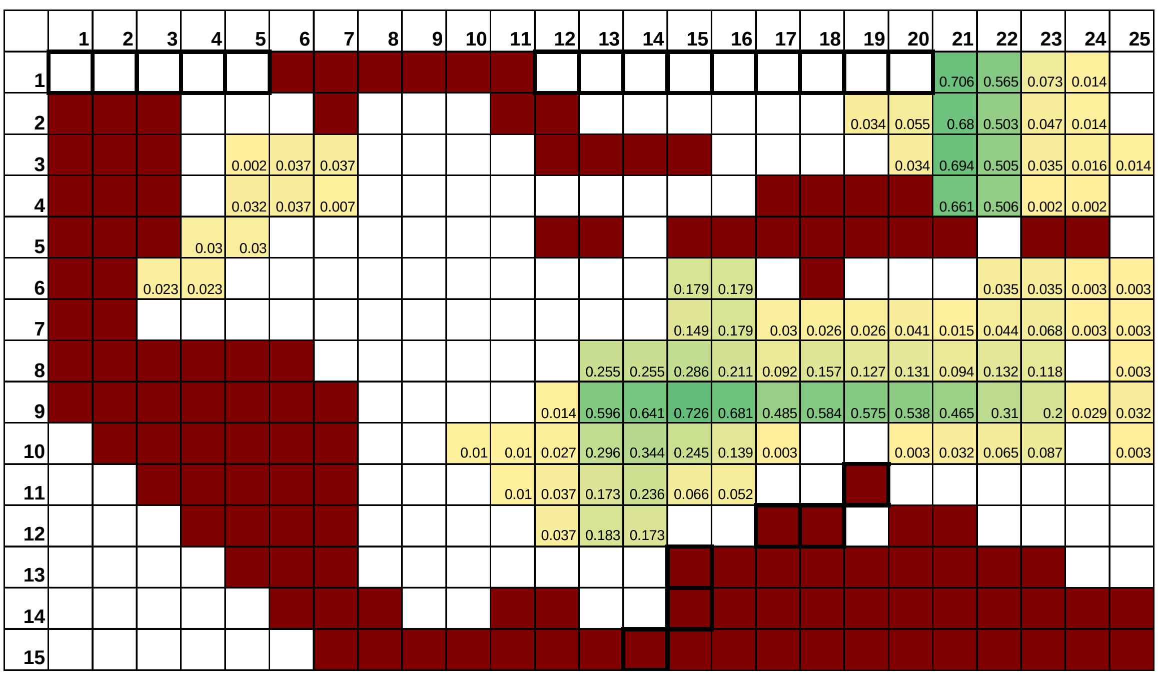


Figure 31. Seaweb Mixed Strategy for Caribbean Scenario with no MPA.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
1																					0.706	0.565	0.073	0.014	
2								0.01								0.01	0.01	0.01	0.044	0.132	1	0.503	0.047	0.014	6E-04
3					0.003	0.047	0.047	0.011	0.002							0.023	0.01	0.01	0.075	0.235	0.694	0.606	0.035	0.016	0.014
4					0.033	0.039	0.019	0.022	0.003	0.001	0.001	0.001	0.001	0.031	0.001	0.004					1	0.506	0.002	0.002	6E-04
5				0.038	0.038	0.005	0.005	0.019	0.009	1E-04	1E-04			0.16								0.063			6E-04
6			0.023	0.023	0.008	0.008	0.027	0.059	0.054	0.072	0.106	0.206	0.206	0.288	0.646	0.343						0.035	0.035	0.06	0.06
7						0.008	0.054	0.081	0.074	0.118	0.159	0.204	0.243	0.243	0.377	0.364	0.038	0.034	0.026	0.041	0.015	0.044	0.068	0.06	0.061
8							0.292	0.247	0.235	0.441	0.275	0.223	0.412	0.374	0.286	0.255	0.143	0.165	0.127	0.131	0.094	0.132	0.118	0.057	0.061
9								0.349	0.364	0.58	0.369	0.364	0.722	0.74	0.726	0.681	0.543	0.648	0.575	0.538	0.465	0.31	0.2	0.086	0.089
10								0.275	0.265	0.36	0.233	0.174	0.325	0.344	0.245	0.145	0.017	0.052	0.044	0.018	0.106	0.065	0.087	0.057	0.061
11								0.043	0.034	0.035	0.04	0.094	0.201	0.236	0.071	0.058	0.008	0.008			0.103	0.035			6E-04
12								0.004	0.004	0.005	0.005	0.123	0.236	0.179	0.006										
13												0.116	0.058	0.031											
14													0.097	0.047											
15																									

Next, to assess the contribution of each platform type, we successively remove each platform type from each region. The results are summarized in Figures 33 and 34. In both regions, we see that removing the MPA has the biggest impact, followed by AEW, Seaweb, frigate and submarine. Given the MPA's effectiveness at both detection and classification it is not surprising that removing the MPA has the biggest impact. The effect of removing the frigate has almost as significant an impact as removing Seaweb in the Pacific. This gap in impact between the frigate and Seaweb is much greater in the Caribbean. The availability and location of deep water is the likely reason. We see in Appendix B that the area adjacent to the entry cells in the Pacific is relatively shallow. This makes Seaweb a less effective detection sensor in an area that would otherwise be an attractive location for Seaweb deployment.

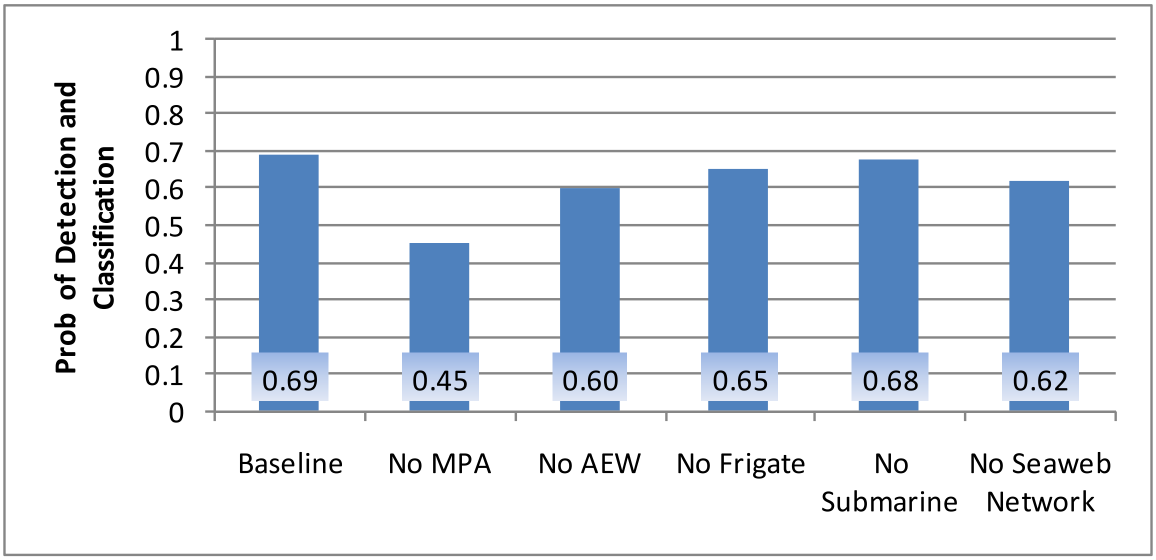


Figure 32. Caribbean Resources Versus Risk. Each bar shows the optimal worst-case probability of detection and classification for the baseline scenario, followed by a sequence of restrictions removing each of the indicated platform types.

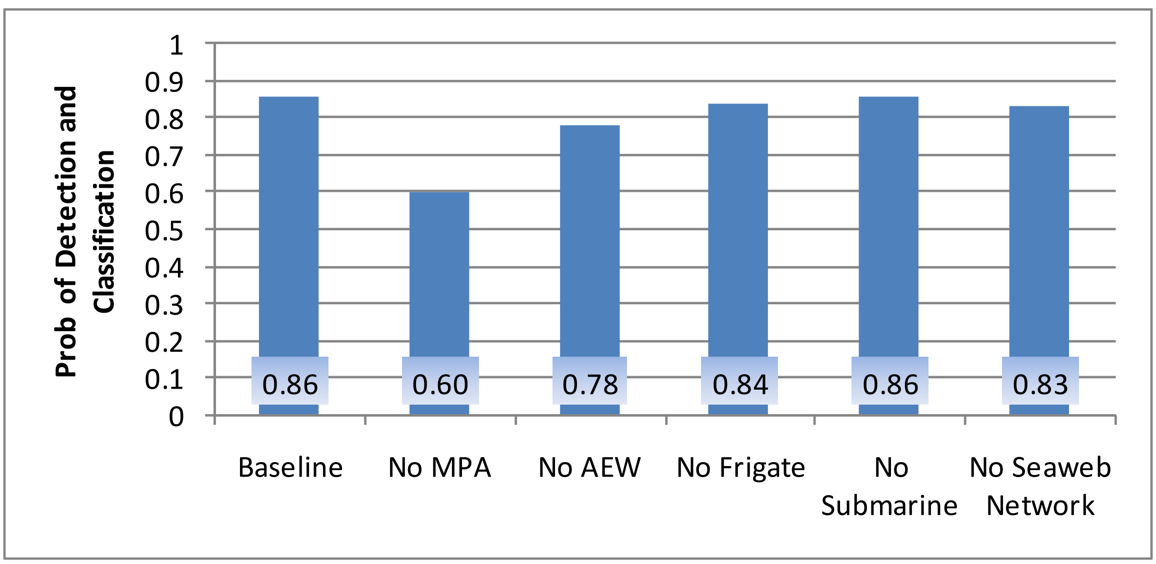


Figure 33. Pacific Resources Versus Risk. Each bar shows the optimal worst-case probability of detection and classification for the baseline scenario, followed by a sequence of restrictions removing each of the indicated platform types.

D IMPROVED INTELLIGENCE

We now modify the Pacific baseline scenario to explore the effect of improved friendly intelligence on attacker goal cells. We model this improved intelligence by reducing the number of goal cells to which the attacker can send his SPSS. As shown in Figure 35, the number of goal cells is now the same and as the number of entry cells.

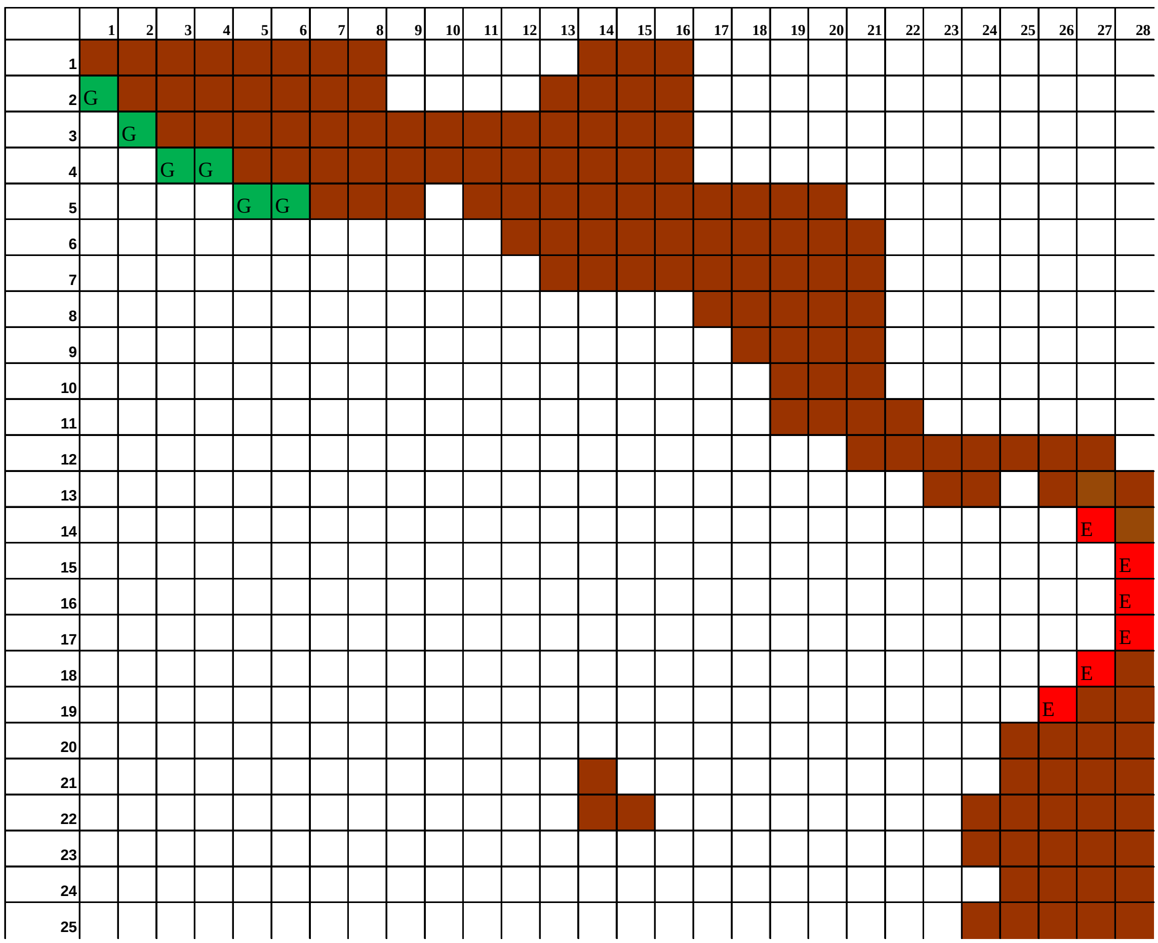


Figure 34. Improved Intelligence Scenario Entry and Goal Cells.

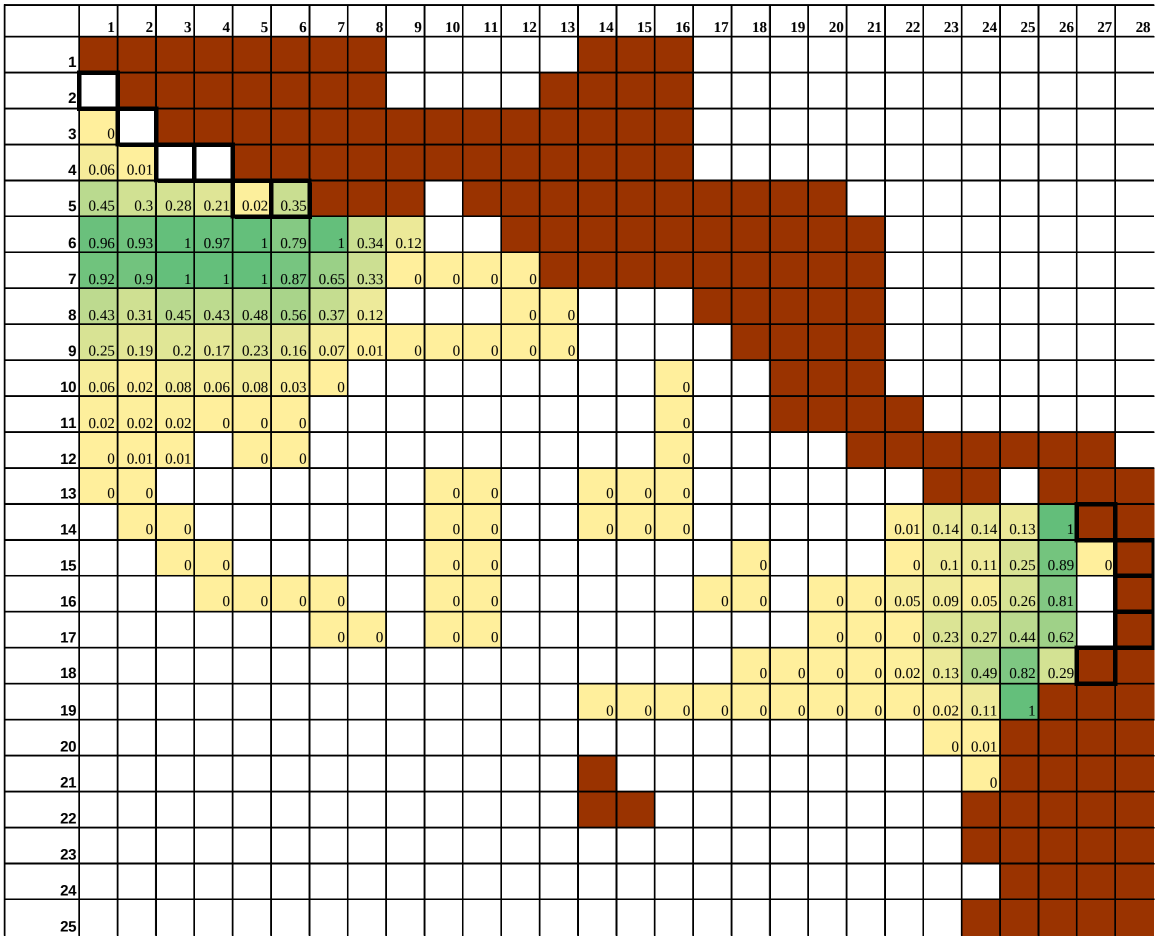


Figure 35. Pacific Improved Intelligence Scenario Overall Mixed Strategy. In contrast to the baseline, having better intelligence that restricts the number of goal cells leads to suggested blockades of both entry and goal cells.

Now that we have an equal number of entry and goal cells, we expend significant effort covering the areas surrounding both the entry and goal cells rather than concentrating most of the effort on the entry cells (Figure 36). The improvement in intelligence improves probability of detection and classification from 0.86 to 0.89. In the baseline scenario the defender is able to effectively cover the entry cells. Given this effectiveness the ability to also cover the goal cells in the improved intelligence scenario only yields a small improvement over the baseline scenario.

E GEOGRAPHIC RESTRICTION

We modify the improved intelligence scenario to restrict the use of MPA and AEW aircraft to waters east of the Galapagos (column j14 and eastward).

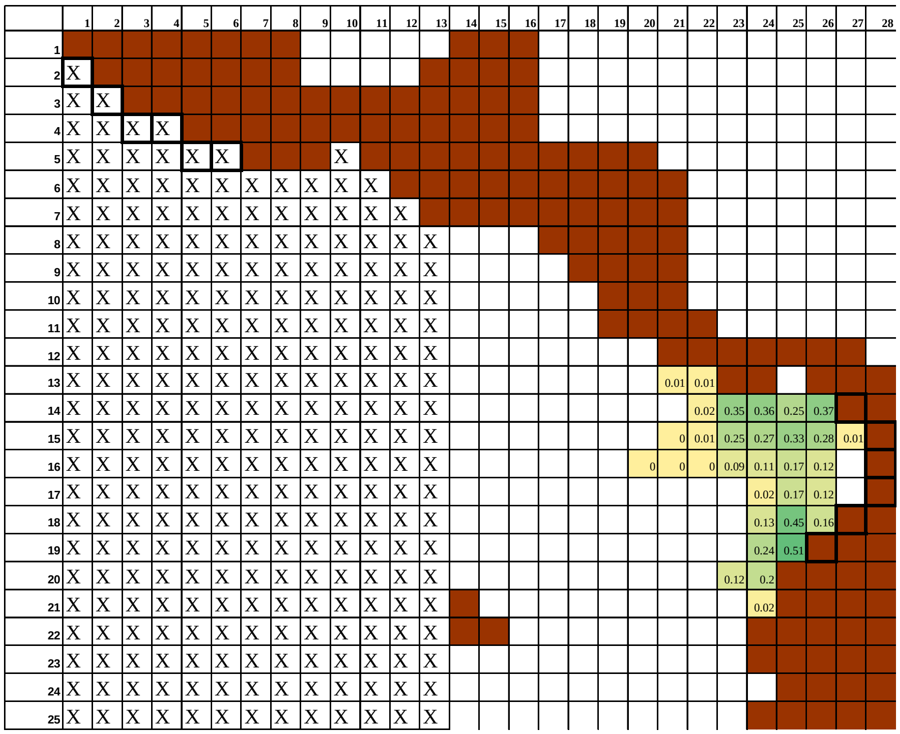


Figure 36. Pacific Geographic Restriction Scenario MPA Mixed Strategy. Areas that cannot be searched by MPA or AEW are marked with an "X."

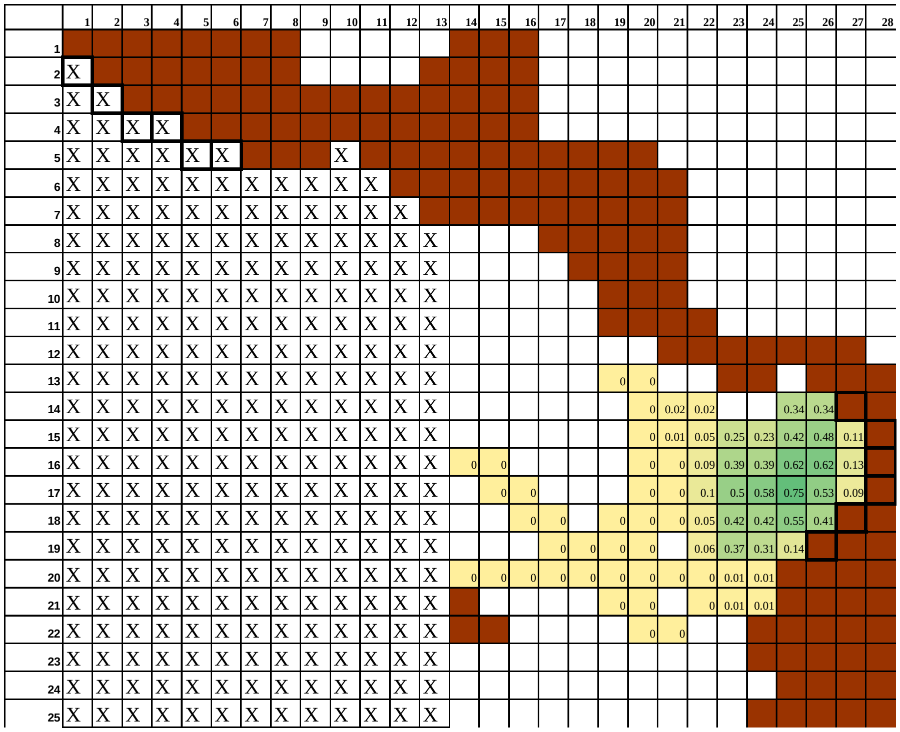


Figure 37. Pacific Geographic Restriction Scenario AEW Mixed Strategy.

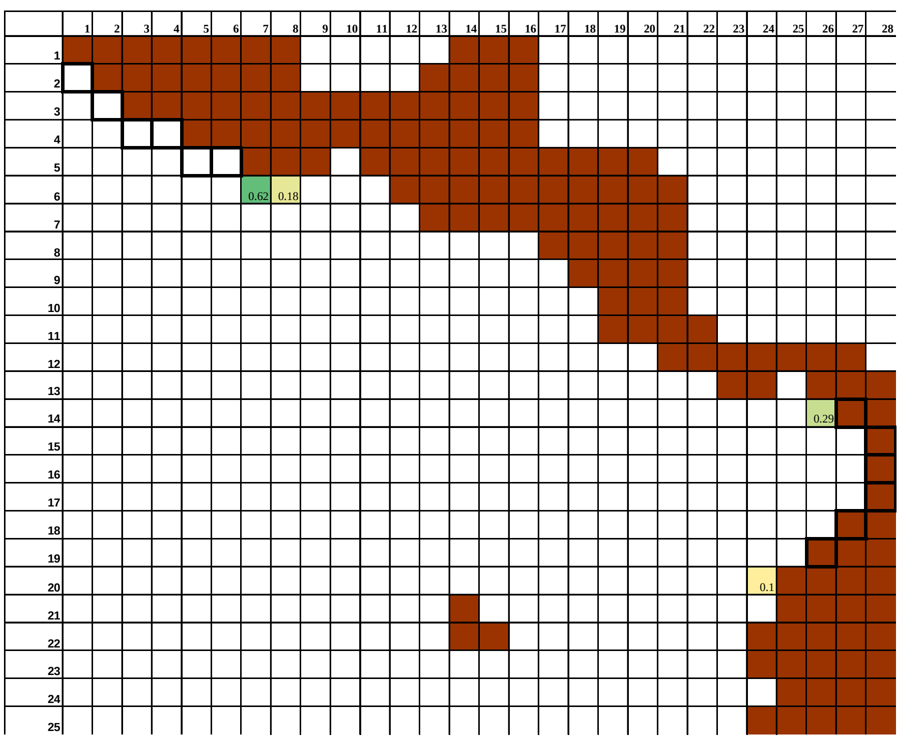


Figure 38. Pacific Geographic Restriction Scenario Frigate Mixed Strategy.

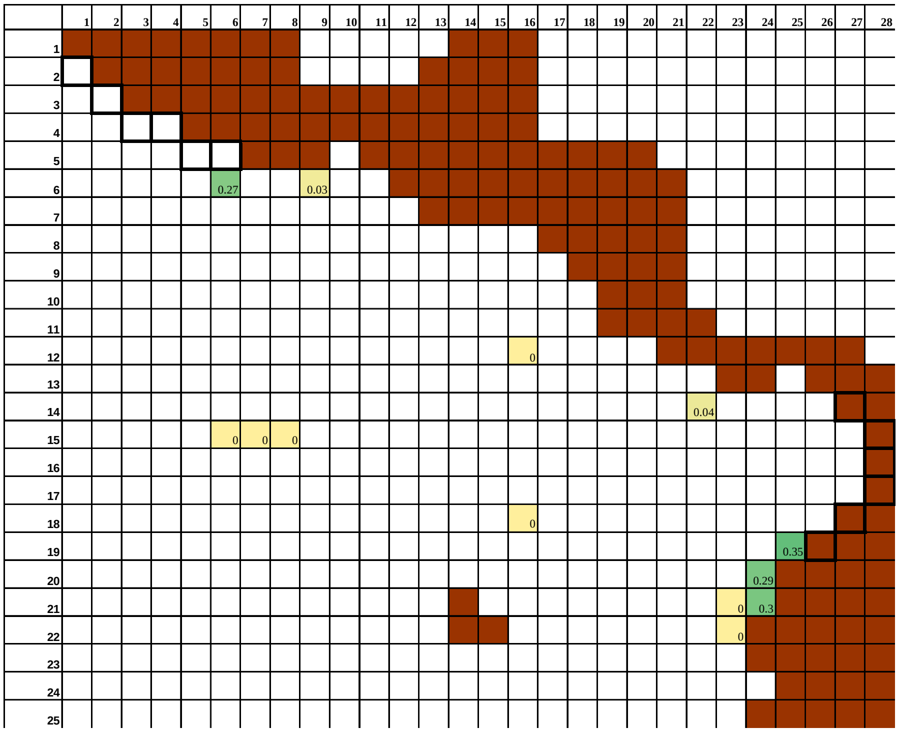


Figure 39. Pacific Geographic Restriction Scenario Submarine Mixed Strategy.

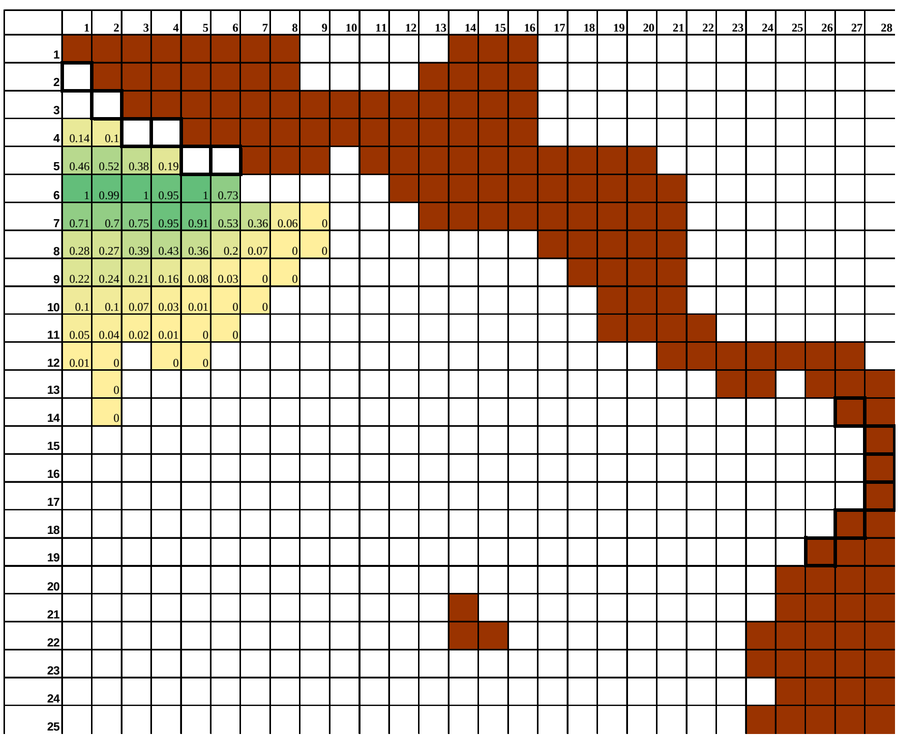


Figure 40. Pacific Geographic Restriction Scenario Seaweb Mixed Strategy.

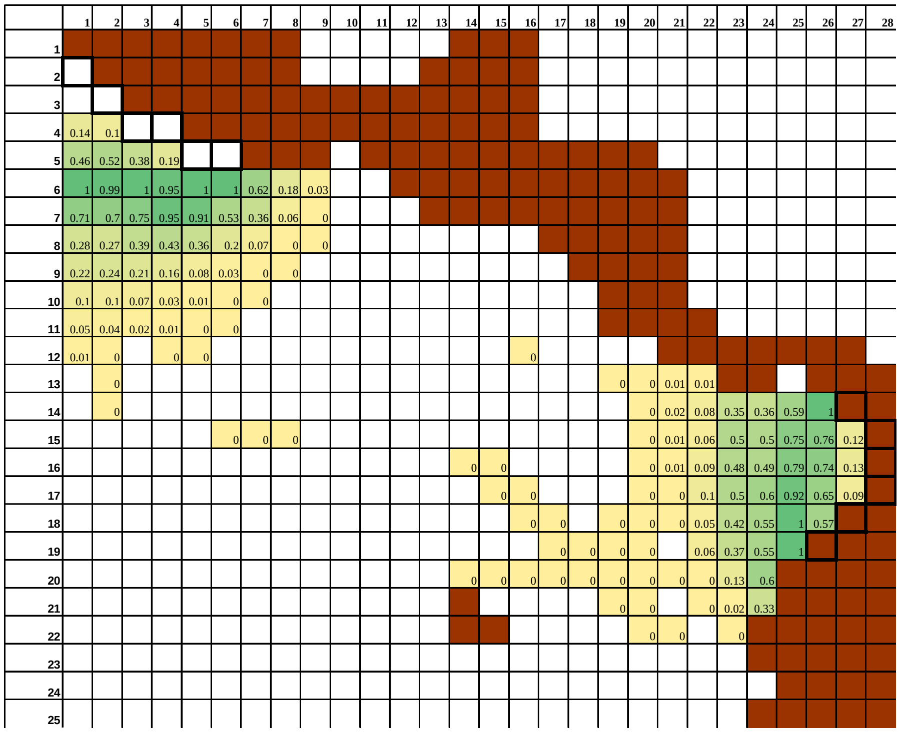


Figure 41. Pacific Geographic Restriction Scenario Overall Mixed Strategy.

We see from Figures 37–42 the defender adjusts to the restriction on AEW and MPA employment by using Seaweb exclusively in cells west of the Galapagos. The defender also sends the frigate west for most of its mixed strategy covering two cells with high shipping density (see Appendix B). The restriction reduces probability of detection and classification from 0.89 to 0.88.

IV. CONCLUSIONS AND RECOMMENDATIONS

This thesis develops an operational level mission-planning aid to counter Self Propelled Semi-Submersibles (SPSS) currently being used by drug smugglers in Colombia to move large volumes of cocaine to the Pacific coast of Mexico, and potentially to the Caribbean coast of the United States. This planning aid suggests optimal placement and disposition of friendly search platforms while considering the intelligent response of the enemy SPSS operators to search efforts. While we agree that a successful operation against an SPSS requires detection, classification, and interdiction, this planning aid addresses only detection and classification, arguably the more technically challenging problems.

The planning aid uses a defender-attacker optimization model to maximize the defender's probability of successful detection and classification of the SPSS through the advantageous disposition of these search platforms against an intelligent attacker operating the semi-submersible. We assume the attacker operating the SPSS has imperfect knowledge of defender platform disposition but is aware that there are defenders that must be avoided. Given this assumption, the solution to the defender-attacker model is a mixed (i.e., probabilistic) strategy for the defender and a least-risk path for the attacker.

We demonstrate our defender-attacker model with both an Eastern Pacific and Caribbean scenarios using five representative platform types whose detection and classification performance varies by platform, and by geography. In all of these cases, we find that our model prescribes a face-valid defensive plan; defenders take advantage of geography by positioning at chokepoints in constrained waterways, and they provide coverage near attacker entry cells and goal cells in the less geographically-constrained scenarios.

We evaluate the effect of removing each class of defender platform on the defender's ability to detect and classify a transiting SPSS. Given our sensor performance assumptions, the removal of the Maritime Patrol Aircraft (MPA) has a much larger effect on detection and classification than removing any other platform. We also demonstrate the value of improved friendly intelligence: in our example our prior knowledge allows focus on fewer goal cells. This improved intelligence yields a corresponding improvement on probability of detection and classification. This also results in the defender re-directing some search resources away from entry cells and toward the now easier-to-cover goal cells. Finally we demonstrate the effect of a geographic restriction on flight operations of the MPA and AEW aircraft. The defender adjusts by sending unrestricted platforms to cover areas that the aircraft cannot.

The model solves our realistic sized scenarios in a few hours using commercially available linear programming optimization software. We recommend further research into developing a heuristic that would solve these problems much faster without requiring licensed optimization software, and preferably on a Navy Marine Corps Intranet (NMCI) computer.

We attempted to run the model assuming perfect attacker knowledge. The resulting mixed integer program is very hard to solve. We recommend further research aimed at improving runtime for this assumption. We additionally recommend research into a tri-level model where the attacker has perfect knowledge of some sensor platform operation plans, but imperfect knowledge of others.

The model ignores interaction between platforms. We recommend further research into a model that improves combined performance when two or more platforms operating in close company.

This thesis contains all data required for the interested reader to reproduce any of the experiments reported.

APPENDIX A. SENSOR PLATFORM PERFORMANCE ASSUMPTIONS AND DATA

This appendix presents the assumptions and data values used to determine both probability of detection and probability of classification. These values will vary for each platform given the searcher's employment decisions.

<u>Indices and index sets [~cardinality]</u>

$c \in C_{ij}$	cells with horizontal, vertical coordinates (alias c1, c2) [~500]
$m \in M$	possible missions [~50,000]
$p \in P$	defending platforms [~10]
$p \in TYPE \subseteq P$	type of defending platform [5]
$m \in M_p \subseteq M$	missions platform p can perform [~10,000]
$c \in C_{pm} \subseteq C$	cells patrolled by platform p while performing mission m [~20]

<u>Data [units]</u>

$cf_{p,m,c}$	coverage factor of platform p, performing mission m, in cell c [nondimensional]
$class_{p,c}$	classification probability given detection of platform p in cell c [probability]
$depth_c$	depth of a cell [nm]
$size_m$	number of cells in mission m [1-20]
$w_{p,c}$	sweep width of a platform p searching in cell c [nm]
v_p	speed of platform p [nm/hr]
n	number of sensors in a Seaweb network [10]

A. MOVING SEARCH PLATFORM TYPES

The majority of the search platform types $p \in TYPE \subseteq P$ move within the cells they are assigned in a mission $c \in C_{p,m} \subseteq C$. These platforms are considered to be searching randomly within these cells. This is a conservative assumption. With w being the sensor sweep width, v being the searcher's speed and t being the time the attacker can transit a cell, and A being the area cell being searched, then by the random search formula (Wagner 1999 p.174):

$$P_d = 1 - e^{-wvt/A}.$$

We call wvt/A the coverage factor (cf). We assume that random search sensors spread their effort evenly among the cells they are assigned. Because all cells total an area 3600 nm^2, A becomes 3600*$size_m$ where $size_m = |C_{pm}|$ is the number of cells assigned in a mission. The SPSS can travel through a cell diagonally or rectilinearly (horizontally or vertically). The transit length is 60 nm for an SPSS transiting rectilinearly. We assume the SPSS speed is a constant 6 knots, so t is 10 hours for an SPSS traveling rectilinearly. Substituting the sweep of the platform in a given cell $w_{p,c}$ and the speed of the platform v_p yields the following coverage factor:

$$cf_{p,m,c} = w_{p,c} v_p * 10 / (3600 * size_m).$$

Which gives the probability of detection of $1 - e^{-cf_{p,m,c}}$ for a rectilinear SPSS transit. For a diagonal transit, we simply multiply $cf_{p,m,c}$ by $\sqrt{2}$. We assume random search platforms ignore any detections outside their assigned search cells. Thus, the probability of detection outside the cells being searched is zero.

1. Maritime Patrol Aircraft (MPA) Platform Type

We assume MPA to search at $v_{maritime\ patrol\ aircraft}$=180 nm/hr. We assume radar is the primary search sensor for MPA. There are several models of MPA employed by Joint Interagency Task Force (JIATF) South with different radars of varying performance. The APS-137 is employed by both USCG C-130's and some USN P-3's and is chosen as the representative MPA radar. Table 2 is used to determine APS-137 sweep width ($w_{maritime\ patrol\ aircraft,c}$) (USCG, 2004, pg. H-55).

Table H-27 Sweep Widths for Forward-Looking Airborne Radar (AN/APS-137)

16 Nautical Mile Radar Range Scale (Sweep Width in Nautical Miles)										
	On-scene Surface Winds (kts)									
Object Type	*< 5*	*to 10*	*to 15*	*to 20*	*to 25*	*to 35*	*to 45*	*to 55*	*to 65*	*> 65*
4 to 10 person life raft	12.1	8.6	3.1	0	0	0	0	0	0	0
17 to 25 foot recreational boat	13.6	11.9	8.2	2.8	0	0	0	0	0	0
26 to 35 foot recreational boat	16.6	16.3	15.4	14.2	12.6	9.5	3.9	0	0	0
36 to 50 foot recreational boat	21.0	20.7	19.9	18.9	17.5	14.7	9.8	3.5	0	0

32 Nautical Mile Radar Range Scale (Sweep Width in Nautical Miles)										
	On-scene Surface Winds (kts)									
Object Type	*< 5*	*to 10*	*to 15*	*to 20*	*to 25*	*to 35*	*to 45*	*to 55*	*to 65*	*> 65*
17 to 25 foot recreational boat	17.4	15.7	12.0	6.6	0	0	0	0	0	0
26 to 35 foot recreational boat	22.1	21.7	20.9	19.7	18.1	14.9	9.3	2.1	0	0
36 to 50 foot recreational boat	29.0	28.7	27.9	26.9	25.5	22.7	17.8	11.5	3.8	0

Table 2. APS-137 Sweep Widths (From USCG, 2004, pg. H-55).

We assume MPA operating in 32nm radar range scale and the SPSS to have roughly the same radar cross section as a 17-25 foot recreational boat. Surface winds will vary from cell to cell based upon the surface wind data presented in Appendix B.

We assume each MPA is capable of employing visual and infra-red (IR) sensors that are organic to the aircraft. Thus, we assign MPA a high probability of classification given detection without the aid of other platforms. We assume each MPA has a probability of classification given detection ($class_{maritime\ patrol\ aircraft,c}$) of 0.95 in cells with sparse shipping density, 0.9 in cells with moderate shipping density and 0.8 in cells with high shipping density.

We assume MPA is able to search up to 10 cells.

2. Airborne Early Warning Aircraft (AEW) Aircraft Platform Type

We assume AEW aircraft to search at $v_{airborne\ early\ warning\ aircraft}$ =180 nm/hr. Radar is the only search sensor for AEW. There are several types of AEW aircraft employed by Joint Interagency Task Force (JIATF) South. The APS-145 is employed by both U.S. Navy E-2's and Customs and Border Patrol P-3's and is chosen as the representative AEW radar. We derive the APS-145 sweep width ($w_{airborne\ early\ warning\ aircraft,c}$) from the APS-137 sweep width. Radar range depends on a number of factors including target radar cross section. APS-145 and APS-137 have different radio frequencies and the radar cross section of a target does vary by radio frequency. However, we assume that this difference in radar cross section is negligible. Transmitted power of APS-137 is approximately 500 kW (Friedman, 2006, pg 210) and APS-145 is approximately 1 mW (Forecast International, 2006). Based on the relative size of the radomes of MPA and AEW versions of the P-3 (Jane's Aircraft Upgrades, 2009) the APS-145 antenna area is roughly 20 times APS-137 antenna area. Because radar range varies by the fourth root of antenna area and transmission power, a factor of 2.5 is applied to the APS-137 sweep width to obtain the estimated APS-145 sweep width (Wagner, 1999, pg 113).

AEW aircraft can only rely on radar for classification and have a relatively poor ability to classify targets. We assume AEW aircraft have a probability of classification given detection ($class_{airborne\ early\ warning\ aircraft,c}$) of 0.5 in cells with sparse shipping density, 0.3 in cells with moderate shipping density and 0.1 in cells with high shipping density.

We assume AEW aircraft are able to search up to 10 cells.

3. Frigate Platform Type

We assume frigates search at $v_{frigate}$ =10 nm/hr. Passive towed array sonar is assumed to be the frigate's primary search sensor. We assume sweep width ($w_{frigate,c}$) of the towed array is 10 nm in cells with poor acoustic conditions, 15 nm with moderate acoustic conditions and 30 nm with good acoustic conditions.

Frigates are equipped with embarked helicopters to aid in classification. Thus, we assume the probability of classification given detection ($class_{frigate,c}$) is 0.95 in cells with sparse shipping density, 0.9 in cells with moderate shipping density, and 0.8 in cells with high shipping density.

We assume frigates are able to search up to three cells.

4. Submarine Platform Type.

Submarines are assumed to be searching at $v_{submarine}$ =10 nm/hr. We assume the hull mounted passive sonar is the submarine's primary search sensor. We assume sweep width ($w_{submarine,c}$) of the towed array is 10 nm in cells with poor acoustic conditions, 15 nm with moderate acoustic conditions and 30nm with good acoustic conditions.

We assume submarines are able to search up to three cells.

Submarines can only rely on acoustic data for classification and have a relatively poor ability to classify targets. We assume submarines have a probability of classification given detection ($class_{submarine,c}$) of 0.5 in cells with sparse shipping density, 0.3 in cells with moderate shipping density and 0.1 in cells with high shipping density.

B. SEAWEB PLATFORM TYPE

Seaweb is a proposed autonomous network of acoustic sensors that communicate with each other through acoustic modems. Data from these sensors is routed to a gateway buoy that communicates to operators through radio or satellite signal. A Seaweb sensor deployed in deep water is expected to have detection ranges of approximately 10 times the water depth ($10*depth_c$) against noisy targets such as SPSS (Rice, 2008). We use the random search formula to calculate the probability of detection. This results in a conservative estimate of the probability of detection. Unlike the moving search sensors the Seaweb network is fixed within a cell once it is installed. As a result the speed of the SPSS is used (v=6 knots) in the calculation. As in the moving search platforms, 10 hours is used for t for rectilinear SPSS transits. We define n as the total number of sensors in a Seaweb network. We assume a Seaweb search sensor's sweep width is $10*depth_c$. Assuming the n sensors in a Seaweb network are spread evenly in the $size_m$ assigned cells results in the following:

$$cf_{seaweb,m,c} = n(10*depth_c)*6*10/(3600*size_m).$$

This gives of probability of detection of $1-e^{-cf_{seaweb,m,c}}$ for a rectilinear SPSS transit. We assume Seaweb sensors ignore any detections outside their assigned search cells. Thus, the probability of detection outside the cells where Seaweb sensors are deployed is zero. We assume Seaweb has a probability of classification given detection ($class_{seaweb,c}$)

of 0.5 in cells with sparse shipping density, 0.3 in cells with moderate shipping density and 0.1 in cells with high shipping density.

We assume the sensors are deployed in a manner that allows communication with a gateway buoy. A Seaweb network can be deployed in as many as n cells. n=10 for all scenarios in this thesis.

APPENDIX B. GEOGRAPHIC MODEL ASSUMPTIONS AND DATA

This appendix presents the assumptions and data used to determine how probability of detection and probability of classification vary by geographic location. We divide seaspace into 60nm by 60nm square cells. The attacker chooses a directed path from an advantageous entry cell node to an advantageous goal cell node. Those cells with an "X" in Figures 43 and 44 depict areas that are non-navigable and cannot be used by the defender and can only be used by the attacker if they are entry cells.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
1						X	X	X	X	X	X														
2	X	X	X				X				X	X													
3	X	X	X									X	X	X	X										
4	X	X	X														X	X	X	X					
5	X	X	X									X	X		X	X	X	X	X	X	X		X	X	
6	X	X																X							
7	X	X																							
8	X	X	X	X	X	X																			
9	X	X	X	X	X	X	X																		
10		X	X	X	X	X	X																		
11			X	X	X	X	X												X						
12				X	X	X	X										X	X		X	X				
13					X	X	X								X	X	X	X	X	X	X	X	X		
14						X	X	X			X	X			X	X	X	X	X	X	X	X	X	X	X
15							X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X

Figure 42. Useable cells in the Caribbean. Cells marked "X" are non-navigable and cannot be used by the defender and can only be used by the attacker if they are entry cells.

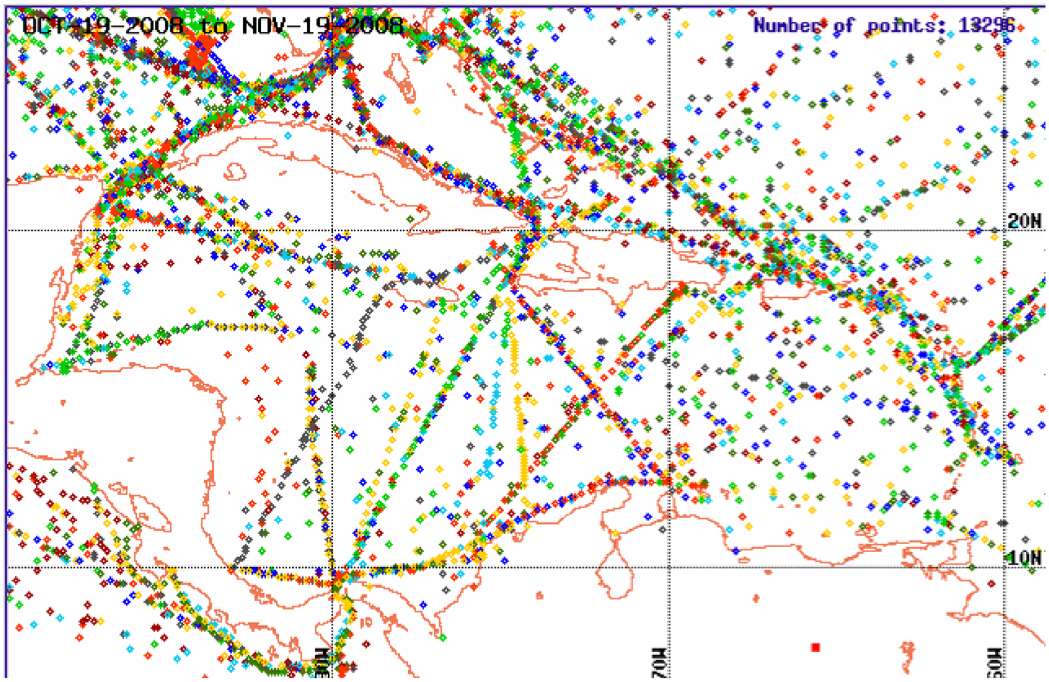
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
1	X	X	X	X	X	X	X	X						X	X	X												
2		X	X	X	X	X	X	X					X	X	X	X												
3			X	X	X	X	X	X	X	X	X	X	X	X	X	X												
4					X	X	X	X	X	X	X	X	X	X	X	X												
5							X	X	X		X	X	X	X	X	X	X	X	X	X								
6												X	X	X	X	X	X	X	X	X	X							
7													X	X	X	X	X	X	X	X	X							
8																	X	X	X	X	X							
9																		X	X	X	X							
10																			X	X	X							
11																			X	X	X	X						
12																					X	X	X	X	X	X	X	
13																							X	X		X	X	X
14																											X	X
15																												X
16																												X
17																												X
18																											X	X
19																										X	X	X
20																									X	X	X	X
21														X											X	X	X	X
22														X	X									X	X	X	X	X
23																								X	X	X	X	X
24																									X	X	X	X
25																								X	X	X	X	X

Figure 43. Useable cells in the Pacific. Cells marked “X” are non-navigable and cannot be used by the defender and can only be used by the attacker if they are entry cells.

A. SHIPPING DENSITY

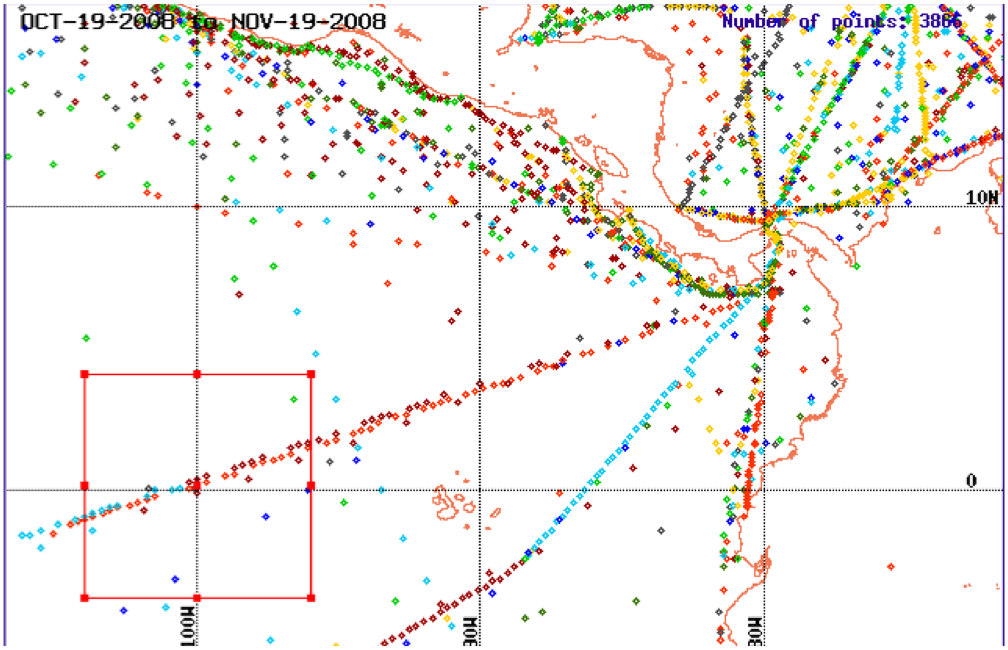
We assume shipping density influences probability of classification. It is more difficult to correctly identify an SPSS if is among other shipping than if it is transiting an area alone. Different types of shipping make classification difficult for different types of sensors. For example, sailboats may look very similar to an SPSS on radar while fishing boats may sound similar to an SPSS on acoustic sensors. We ignore the effect of different types of shipping by assuming that the shipping densities we use convert to an equal number of ships that might confuse radar or acoustic sensors. Figures 45 and 46 show a one-month plot of ships that report metrological data to the National Oceanic and Atmospheric Administration (NOAA) through the Shipboard Environmental data

Acquisition System (SEAS) that we use to decide which cells have high, moderate, or sparse shipping densities (NOAA, 2008). We then assign a 3 for high shipping density 2 for moderate and 1 for sparse in the shipping density model.



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
1	1	1	2	3	3	X	X	X	X	X	X	3	1	1	1	2	1	1	1	3	1	1	1	1	1
2	X	X	X	3	3	3	X	1	1	1	X	X	3	3	3	2	1	1	1	1	3	1	1	1	1
3	X	X	X	3	1	2	1	1	1	1	1	X	X	X	X	3	1	1	1	1	1	3	1	1	1
4	X	X	X	3	1	1	2	2	2	2	1	2	2	2	3	1	X	X	X	X	1	2	3	3	1
5	X	X	X	2	1	1	1	1	1	1	2	X	X	3	X	X	X	X	X	X	X	2	X	X	3
6	X	X	2	1	1	1	1	1	1	1	2	1	1	1	2	1	1	X	1	2	2	1	1	1	1
7	X	X	2	2	2	2	2	2	2	2	1	1	1	1	1	2	1	1	1	2	1	1	1	1	1
8	X	X	X	X	X	X	1	1	1	2	1	1	1	1	1	1	2	1	2	1	1	1	1	1	1
9	X	X	X	X	X	X	X	1	1	2	1	1	1	1	1	1	1	2	1	1	1	1	1	1	1
10		X	X	X	X	X	X	1	1	2	1	1	1	1	1	1	2	1	2	1	1	1	1	1	1
11			X	X	X	X	X	1	2	1	2	1	1	1	1	2	2	2	X	2	1	1	1	1	1
12				X	X	X	X	1	2	1	2	1	2	2	2	3	X	X	1	X	X	1	1	1	1
13					X	X	X	2	2	2	3	2	1	1	X	X	X	X	X	X	X	X	X	1	1
14						X	X	X	1	1	X	X	1	1	X	X	X	X	X	X	X	X	X	X	X
15							X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X

Figure 44. SEAS Data and Corresponding Caribbean Shipping Density Model. Densities are scored with discrete values from 1 (low) to 3 (high) (From NOAA, 2008).



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
1	X	X	X	X	X	X	X	X						X	X	X												
2	2	X	X	X	X	X	X	X					X	X	X	X												
3	1	2	X	X	X	X	X	X	X	X	X	X	X	X	X	X												
4	1	1	3	3	X	X	X	X	X	X	X	X	X	X	X	X												
5	1	1	1	1	3	3	X	X	X	1	X	X	X	X	X	X	X	X	X	X								
6	1	1	1	1	1	1	3	3	1	2	2	X	X	X	X	X	X	X	X	X	X							
7	1	1	1	1	1	1	1	1	2	2	1	2	X	X	X	X	X	X	X	X	X							
8	1	1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	X	X	X	X	X							
9	1	1	1	1	1	1	1	1	1	1	1	1	2	2	1	1	2	X	X	X	X							
10	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	2	1	2	X	X	X							
11	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	3	X	X	X	X						
12	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	3	3	X	X	X	X	X	X	X	
13	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	3	3	X	X	3	X	X	X
14	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	3	3	3	1	X	X
15	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	2	2	2	3	1	1	X
16	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	2	1	1	2	1	2	1	1	X
17	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	2	2	1	1	1	2	1	1	2	1	1	X
18	1	1	1	1	1	1	1	1	1	1	1	1	2	2	2	1	1	1	1	1	2	1	1	1	2	1	X	X
19	1	1	1	1	1	1	1	1	1	2	2	2	1	1	1	1	1	1	1	1	2	1	1	2	1	X	X	X
20	1	1	1	1	1	1	2	2	2	1	1	1	1	1	1	1	1	1	1	2	1	1	1	2	X	X	X	X
21	1	1	1	2	2	2	1	1	1	1	1	1	1	X	1	1	1	1	2	1	1	1	1	2	X	X	X	X
22	1	2	2	1	1	1	1	1	1	1	1	1	1	X	X	1	1	2	1	1	1	1	1	X	X	X	X	X
23	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	1	1	1	1	1	1	X	X	X	X	X
24	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	1	1	1	1	1	1	1	1	X	X	X	X
25	1	1	1	1	1	1	1	1	1	1	1	1	1	2	2	1	1	1	1	1	1	1	1	X	X	X	X	X

Figure 45. SEAS Data and Corresponding Pacific Shipping Density Model. Density scores vary from 1 (low) to 3 (high) (From NOAA, 2008).

B. SURFACE WINDS

As discussed in Appendix A, surface winds influence radar sweep widths. We use a 10-day average of surface winds taken from Special Sensor Microwave/Imager (SSM/I) satellite data as a representative wind distribution for both the Caribbean and Pacific (NOAA, 2009).

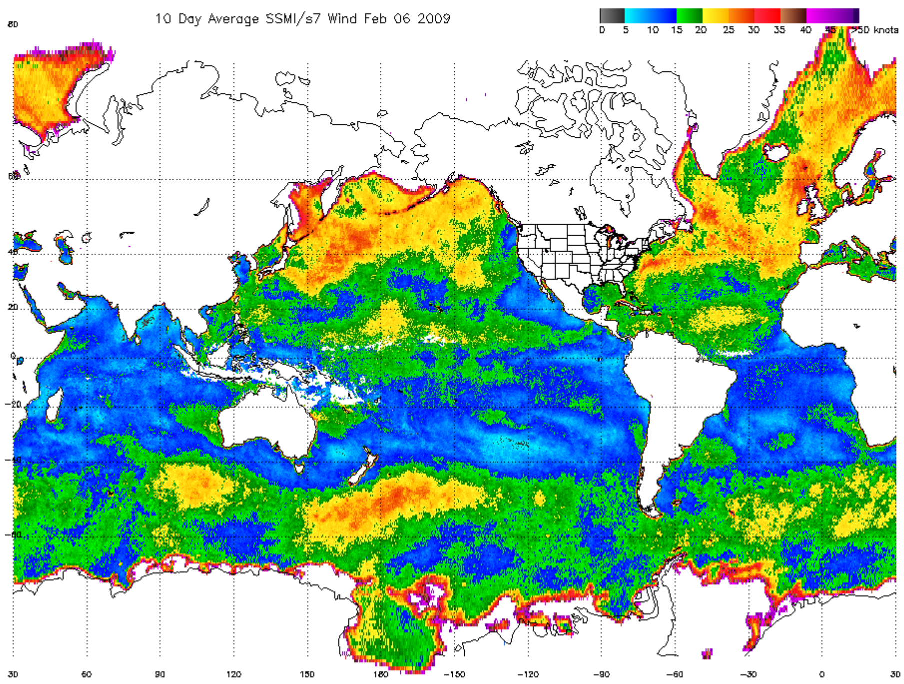


Figure 46. 10-Day Average Wind February 06 2009 (From: NOAA, 2009)..

We use the 10-day average winds from Figure 47 to construct the wind models shown in Figures 48 and 49.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
1	15	15	20	20	15	X	X	X	X	X	X	25	15	15	15	20	20	20	15	15	10	10	15	15	15
2	X	X	X	20	15	15	X	15	15	15	X	X	15	25	15	15	15	15	15	15	10	10	15	15	15
3	X	X	X	15	10	15	10	15	15	15	20	X	X	X	X	15	15	15	15	15	10	10	10	15	15
4	X	X	X	15	15	15	15	15	15	15	15	20	15	10	20	20	X	X	X	X	15	15	20	15	15
5	X	X	X	10	15	15	15	15	15	15	20	X	X	10	X	X	X	X	X	X	X	15	X	X	15
6	X	X	15	15	15	15	15	15	15	15	10	20	15	15	15	15	20	X	20	15	15	15	25	15	15
7	X	X	15	20	20	20	10	15	15	15	10	15	15	15	15	15	15	15	15	15	15	15	15	15	15
8	X	X	X	X	X	X	10	15	15	15	15	15	15	15	20	15	15	20	20	20	15	15	15	15	15
9	X	X	X	X	X	X	X	15	15	15	15	15	15	15	20	20	15	15	20	20	20	20	15	15	15
10		X	X	X	X	X	X	15	15	15	20	15	15	20	20	20	15	20	20	20	20	20	15	15	15
11			X	X	X	X	X	15	15	15	15	15	20	20	15	15	15	15	X	20	25	20	15	15	15
12				X	X	X	X	10	15	15	15	15	15	20	15	25	X	X	15	X	X	20	20	15	15
13					X	X	X	10	10	10	15	15	15	20	X	X	X	X	X	X	X	X	X	15	15
14						X	X	X	10	15	X	X	25	25	X	X	X	X	X	X	X	X	X	X	X
15							X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X

Figure 47. Caribbean Wind Model (average wind in knots).

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
1	X	X	X	X	X	X	X	X						X	X	X												
2	10	X	X	X	X	X	X	X					X	X	X	X												
3	10	10	X	X	X	X	X	X	X	X	X	X	X	X	X	X												
4	10	10	20	10	X	X	X	X	X	X	X	X	X	X	X	X												
5	10	10	10	5	5	10	X	X	X	25	X	X	X	X	X	X	X	X	X	X								
6	10	5	5	5	5	5	10	10	20	20	10	X	X	X	X	X	X	X	X	X	X							
7	10	10	10	10	5	10	10	10	20	10	10	10	X	X	X	X	X	X	X	X	X							
8	10	1	10	10	10	10	10	15	15	15	10	10	10	10	10	20	X	X	X	X	X							
9	10	10	10	10	10	10	15	15	15	15	10	10	10	10	10	10	20	X	X	X	X							
10	15	15	15	15	15	15	15	15	15	15	10	10	15	15	10	15	15	20	X	X	X							
11	15	15	15	15	15	15	15	15	15	15	10	10	15	15	10	10	15	20	X	X	X	X						
12	15	15	15	15	15	15	15	10	10	10	10	10	10	10	10	10	5	10	10	10	X	X	X	X	X	X	X	
13	15	15	15	15	15	15	15	10	10	10	10	10	10	10	10	10	10	5	5	10	10	15	X	X	20	X	X	X
14	15	15	15	15	15	10	10	10	10	10	10	5	5	10	10	10	10	10	5	10	10	15	15	15	10	10	X	X
15	10	10	10	10	10	10	10	10	10	10	10	5	5	5	10	10	10	10	5	10	10	10	10	15	10	5	10	X
16	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	5	10	10	10	10	10	10	5	10	X
17	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	X
18	10	10	10	10	10	10	10	10	10	5	10	10	10	10	10	10	10	10	10	10	10	5	5	10	10	10	X	X
19	10	10	10	5	5	10	10	5	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	X	X	X
20	10	5	5	5	5	5	5	5	5	5	10	10	10	10	10	10	10	10	10	10	5	10	10	10	X	X	X	X
21	10	5	5	5	10	5	5	5	5	5	5	10	10	X	15	10	5	5	5	5	10	10	10	10	X	X	X	X
22	5	5	10	10	10	10	10	10	5	10	10	10	10	X	X	15	5	5	5	5	10	10	10	X	X	X	X	X
23	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	5	5	5	5	5	5	5	10	X	X	X	X	X
24	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	X	X	X	X
25	10	15	15	15	15	10	10	15	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	X	X	X	X	X

Figure 48. Pacific Wind Model (average wind in knots).

C. WATER DEPTH

As discussed in Appendix A, water depth influences Seaweb sensor performance. Water depth data from Google Earth 5.0 is used to build the water depth models in Figures 50 and 51 (Google Earth, 2009).

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
1	1	9	1	94	38	X	X	X	X	X	X	7	14	30	49	35	82	148	178	180	178	185	187	188	189
2	X	X	X	75	139	144	X	132	13	11	X	X	40	56	28	5	88	145	182	177	183	180	176	186	184
3	X	X	X	3	110	141	102	105	74	36	24	X	X	X	X	25	1	23	70	16	251	237	252	224	225
4	X	X	X	144	126	130	105	90	36	61	102	83	89	88	23	4	X	X	X	X	118	94	102	83	71
5	X	X	X	142	88	94	109	112	71	73	51	X	X	45	X	X	X	X	X	X	X	11	X	X	1
6	X	X	117	117	90	63	56	23	24	22	5	33	43	58	97	128	95	X	129	139	14	138	132	112	57
7	X	X	10	12	5	3	1	3	4	28	27	53	67	79	97	126	119	100	127	133	138	148	158	151	135
8	X	X	X	X	X	X	1	9	34	58	83	95	111	122	136	117	108	127	133	136	146	159	157	140	102
9	X	X	X	X	X	X	X	2	44	73	98	125	131	132	138	129	116	125	133	135	146	159	157	141	106
10		X	X	X	X	X	X	8	53	100	95	121	120	126	130	104	67	43	43	80	83	149	165	131	100
11			X	X	X	X	X	30	36	84	107	104	111	107	105	85	50	10	X	5	16	19	100	51	78
12				X	X	X	X	56	77	92	102	105	112	98	14	3	X	X	1	X	X	6	38	14	2
13					X	X	X	6	53	52	52	69	68	3	X	X	X	X	X	X	X	X	X	1	1
14						X	X	X	11	6	X	X	4	10	X	X	X	X	X	X	X	X	X	X	X
15							X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X

Figure 49. Caribbean Water Depth Model (average depth in hundreds of feet).

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
1	X	X	X	X	X	X	X	X						X	X	X												
2	28	X	X	X	X	X	X	X					X	X	X	X												
3	85	86	X	X	X	X	X	X	X	X	X	X	X	X	X	X												
4	97	103	131	96	X	X	X	X	X	X	X	X	X	X	X	X												
5	98	118	122	106	74	57	X	X	X	2	X	X	X	X	X	X	X	X	X	X								
6	119	109	112	112	112	115	106	65	83	50	20	X	X	X	X	X	X	X	X	X	X							
7	93	103	108	110	110	109	113	120	133	148	117	36	X	X	X	X	X	X	X	X	X							
8	98	101	107	110	109	113	124	134	135	139	115	104	98	68	44	1	X	X	X	X	X							
9	99	103	107	110	110	120	132	135	136	133	131	127	130	128	96	54	22	X	X	X	X							
10	99	104	108	112	116	126	131	132	134	133	130	128	123	119	115	108	65	15	X	X	X							
11	99	110	114	117	124	130	132	131	129	127	128	129	123	121	119	114	98	55	X	X	X	X						
12	100	104	110	114	119	124	126	129	125	123	120	118	117	118	116	114	107	98	51	38	X	X	X	X	X	X	X	
13	103	99	103	109	114	119	120	125	121	119	116	113	117	118	114	114	108	99	88	77	50	7	X	X	9	X	X	X
14	109	100	103	109	113	115	118	120	120	119	117	116	119	120	114	113	109	102	89	61	54	61	31	51	88	109	X	X
15	110	105	107	110	114	115	117	119	119	119	117	108	115	118	113	94	88	78	59	89	90	90	35	60	99	115	35	X
16	113	107	108	108	109	107	108	116	116	114	111	104	103	108	104	85	74	57	61	96	103	102	55	54	96	112	36	X
17	116	109	108	106	102	89	96	111	112	109	104	103	98	92	87	74	60	55	83	96	106	108	109	78	99	108	14	X
18	113	108	108	109	106	106	108	111	106	98	88	89	85	77	71	68	68	82	92	109	115	110	106	121	140	6	X	X
19	113	110	112	112	111	111	109	105	100	93	84	86	76	69	68	75	83	88	95	104	113	112	102	113	56	X	X	X
20	114	110	111	113	111	112	106	103	102	94	88	88	79	52	56	66	84	88	93	98	105	102	86	22	X	X	X	X
21	116	110	107	104	97	106	109	108	108	108	106	98	52	X	31	62	80	86	91	99	93	73	51	21	X	X	X	X
22	115	108	105	106	108	110	110	109	109	103	97	107	41	X	X	10	51	73	85	72	58	59	10	X	X	X	X	X
23	116	108	107	108	110	113	113	112	114	108	100	109	110	110	55	63	60	82	93	87	95	73	96	X	X	X	X	X
24	116	108	108	110	112	115	117	118	119	111	107	111	111	108	104	107	106	104	106	103	103	101	133	40	X	X	X	X
25	109	113	110	109	118	116	122	120	117	119	121	121	121	120	115	115	116	119	112	110	112	117	70	X	X	X	X	X

Figure 50. Pacific Water Depth Model (average depth in hundreds of feet).

D. ACOUSTIC CONDITIONS

The performance of passive acoustic sensors such as a towed-array sonar or a submarine's hull-mounted sonar depend on a number of factors. We only consider a few of these to help build a representative distribution of cells with good, moderate and poor acoustic conditions for both of our geographic areas of interest. The factors considered are shipping density, wind, and water depth. Higher shipping density results in higher ambient noise that degrades acoustic sensor performance. Higher winds lead to higher sea states that also increase the ambient noise. Shallow water has less favorable acoustic conditions than deeper water. In order to combine all these factors, we assign a 1, 2 or 3 for each factor. We have already expressed shipping density in this manner. For wind, we assign 1 for average winds less than or equal to 10 knots, a 2 for winds greater than 10 knots and less than or equal to 20 knots, and 3 for winds greater than 20 knots. For water depth, we assign 1 for average water depth greater than or equal to 5,000 feet, a 2 for water depth less than 5,000 and greater than or equal to 1,000 feet, and 3 for water depth less than 1,000 feet. We add these three factors to obtain the acoustic condition models in Figures 52 and 53. We consider numbers in the range 3–5 to represent good acoustic conditions, 6–7 to represent moderate acoustic conditions, and 8–9 to represent poor acoustic conditions.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
1	6	6	8	7	7	X	X	X	X	X	X	9	5	5	5	7	5	5	4	6	3	3	4	4	4
2	X	X	X	7	6	6	X	4	5	5	X	X	7	7	7	7	4	4	4	4	5	3	4	4	4
3	X	X	X	8	3	5	3	4	4	5	6	X	X	X	X	7	6	5	4	5	3	5	3	4	4
4	X	X	X	6	4	4	5	5	6	5	4	6	5	4	8	7	X	X	X	X	4	5	7	6	4
5	X	X	X	4	4	4	4	4	4	4	6	X	X	6	X	X	X	X	X	X	X	6	X	X	8
6	X	X	5	4	4	4	4	5	5	5	6	6	5	4	5	4	5	X	5	5	6	4	5	4	4
7	X	X	7	7	8	8	6	7	7	6	4	4	4	4	4	5	4	4	4	5	4	4	4	4	4
8	X	X	X	X	X	X	5	6	5	5	4	4	4	4	5	4	5	5	6	5	4	4	4	4	4
9	X	X	X	X	X	X	X	6	5	5	4	4	4	4	5	5	4	5	5	5	5	5	4	4	4
10		X	X	X	X	X	X	6	4	5	5	4	4	5	5	5	5	6	7	5	5	5	4	4	4
11			X	X	X	X	X	5	6	4	5	4	5	5	4	5	5	7	X	8	6	6	4	4	4
12				X	X	X	X	3	5	4	5	4	5	6	6	9	X	X	6	X	X	7	6	5	6
13					X	X	X	6	4	4	6	5	4	7	X	X	X	X	X	X	X	X	X	6	6
14						X	X	X	4	6	X	X	7	7	X	X	X	X	X	X	X	X	X	X	X
15							X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X

Figure 51. Caribbean Acoustic Condition Model. Cell entries in the range 3–5 represent good, 6–7 moderate, and 8–9 poor conditions.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
1	X	X	X	X	X	X	X	X						X	X	X												
2	5	X	X	X	X	X	X	X					X	X	X	X												
3	3	4	X	X	X	X	X	X	X	X	X	X	X	X	X	X												
4	3	3	7	5	X	X	X	X	X	X	X	X	X	X	X	X												
5	3	3	3	3	5	5	X	X	X	7	X	X	X	X	X	X	X	X	X	X								
6	3	3	3	3	3	3	5	5	5	6	5	X	X	X	X	X	X	X	X	X	X							
7	3	3	3	3	3	3	3	3	6	4	3	5	X	X	X	X	X	X	X	X	X							
8	3	3	3	3	3	3	3	4	4	4	4	4	4	4	5	8	X	X	X	X	X							
9	3	3	3	3	3	3	4	4	4	4	3	3	4	4	3	3	7	X	X	X	X							
10	4	4	4	4	4	4	4	4	4	4	3	3	4	4	4	5	4	7	X	X	X							
11	4	4	4	4	4	4	4	4	4	4	3	3	4	4	3	3	5	7	X	X	X	X						
12	4	4	4	4	4	4	4	3	3	3	3	3	3	3	3	3	3	3	5	6	X	X	X	X	X	X	X	
13	4	4	4	4	4	4	4	3	3	3	3	3	3	3	3	3	3	3	3	3	5	8	X	X	9	X	X	X
14	4	4	4	4	4	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	4	7	6	5	3	X	X
15	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	4	4	5	5	5	3	4	X
16	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	4	4	3	3	4	3	4	3	4	X
17	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	4	4	4	3	3	3	4	3	3	4	3	4	X
18	3	3	3	3	3	3	3	3	3	3	3	3	4	4	4	3	3	3	3	3	4	3	3	3	4	5	X	X
19	3	3	3	3	3	3	3	3	3	4	4	4	3	3	3	3	3	3	3	3	4	3	3	4	3	X	X	X
20	3	3	3	3	3	3	4	4	4	3	3	3	3	3	3	3	3	3	3	4	3	3	3	5	X	X	X	X
21	3	3	3	4	4	4	3	3	3	3	3	3	3	X	5	3	3	3	4	3	3	3	3	5	X	X	X	X
22	3	4	4	3	3	3	3	3	3	3	3	3	4	X	X	6	3	4	3	3	3	3	5	X	X	X	X	X
23	4	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	4	3	3	3	3	3	3	X	X	X	X	X
24	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	4	3	3	3	3	3	3	3	4	X	X	X	X
25	3	4	4	4	4	3	3	4	3	3	3	3	3	4	4	3	3	3	3	3	3	3	3	X	X	X	X	X

Figure 52. Pacific Acoustic Condition Model. Cell entries in the range 3-5 represent good, 6-7 moderate, and 8-9 poor conditions.

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